K. D. K. College of Engineering, Nagpur
Department of Mechanical Engineering
Computer Aided Design
Unit – V: Trusses (Plane Truss)

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Introduction:
1) The members of truss are subjected to the direct tension or compression.
2) All loads and reactions are applied only at the joints.
3) FEM is applicable to statically determinate of indeterminate structures.
4) FEM also provides joint deflections, effect of temperature change and reactions at the support.

Local and Global Coordinate Systems:

The main difference between the one-dimensional structures and trusses is that the elements of a truss have various orientations. To account for these different orientations, local and global coordinate systems are introduced.

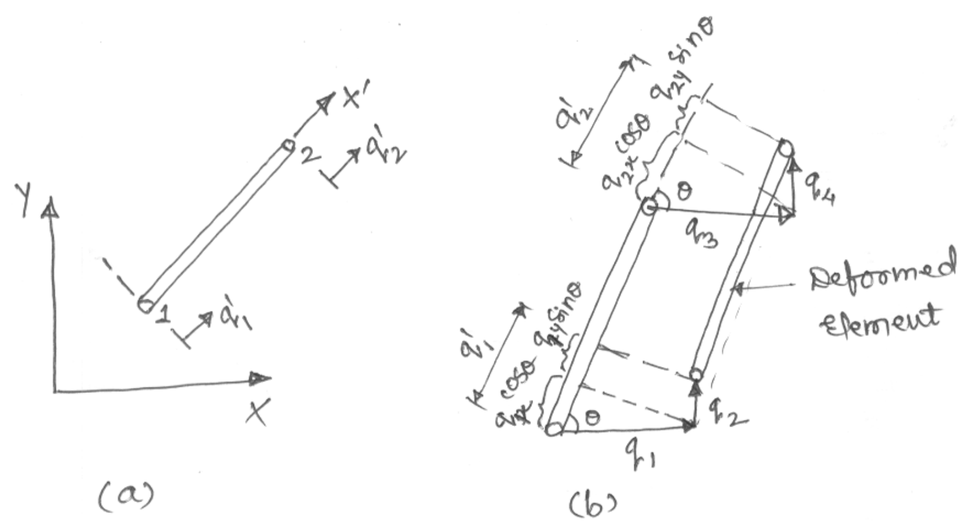

Fig.: A two dimensional truss element (a) in a local coordinate system and (b) in a global coordinate system

Let q_1' and q_2' be the displacements of nodes 1 and 2 respectively in the local coordinate system.

The element displacement vector in the local coordinate system is,

$q' = [q_1', q_2']^T$

The element displacement vector in the global coordinate system is a (4×1) vector denoted by,

$q = [q_{1x}, q_{1y}, q_{2x}, q_{2y}]^T$

The relationship between q' and q is developed as follows,

$q_1' = q_{1x} cos\theta + q_{1y} sin\theta$

$q_2' = q_{2x} cos\theta + q_{2y} sin\theta$

The direction cosines 'l' and 'm' are introduced as,

$l = cos\theta; m = sin\theta$

These direction cosines are the cosines of the angles that the local $X' - axis$ makes with the global X & Y axis respectively.

$\therefore q_1' = q_{1x} l + q_{1y} m$

$q_2' = q_{2x} l + q_{2y} m$

$$\begin{bmatrix} q_1' \\ q_2' \end{bmatrix} = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{2x} \\ q_{2y} \end{bmatrix}$$

$$q' = Lq$$

Where,

L = Transformation Matrix

$$L = \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

Note:-

Que. What do you mean by transformation matrix? Derive the transformation matrix for truss element. (5m, S-03)

Direction Cosines (l & m)

Direction cosines are calculated from the nodal data.

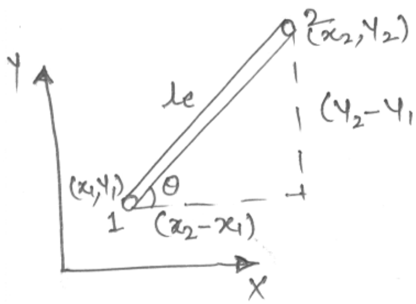

Fig.

$$l = cos\theta; m = sin\theta$$

OR

$$l = \frac{x_2 - x_1}{l_e}; m = \frac{y_2 - y_1}{l_e}$$

$$l_e = \sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$$

Stiffness Matrix of Truss Element

The truss element is a 1-D element when viewed in the local coordinate system. The element stiffness matrix for a truss element in the local coordinate system is given by,

$$K' = \frac{A_e E_e}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad (1)$$

Where,

A_e – Elemental cross-section area

E_e – Young's Modulus

l_e – Elemental length

The element strain energy in local coordinate is given by,

$$U_e = \frac{1}{2} q'^T K' q' \qquad (2)$$

Where,

$q' = Lq$

$q'^T = L^T q^T$

$\therefore U_e = \frac{1}{2} q^T [L^T K' L] q$

The strain energy in global coordinate system can be written as,

$U_e = \frac{1}{2} q^T K q$

Where,

K is the element stiffness matrix in global coordinates.

$K = L^T K' L$

$$K = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \frac{A_e E_e}{l_e} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix}$$

$$K = \begin{bmatrix} l & 0 \\ m & 0 \\ 0 & l \\ 0 & m \end{bmatrix} \frac{A_e E_e}{l_e} \begin{bmatrix} l & m & -l & -m \\ -l & -m & l & m \end{bmatrix}$$

$$\therefore K = \frac{A_e E_e}{l_e} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Que.

1) Derive an expression of element stiffness matrix in two-dimensional truss element. **[S-07, 3m]**

2) What is transformation matrix for truss element? Find out element stiffness matrix of truss element with the help of stiffness matrix of 1-D element. **[W-01, 6m], [S-01, 8m]**

Elemental Stress Calculation

A truss element in local coordinates is a simple two-force member. Thus, the stress 'σ' in a truss element is given by,

$\sigma = E_e \epsilon$

Since the strain $'\epsilon'$ is the change in length per unit original length,

$$\sigma = E_e \frac{q_2' - q_1'}{l_e}$$

$$\sigma = \frac{E_e}{l_e} [-1 \;\; 1] \begin{Bmatrix} q_1' \\ q_2' \end{Bmatrix}$$

This equation can be written in terms of the global displacement 'q' using the transformation

$q' = Lq$, as

$$\sigma = \frac{E_e}{l_e} [-1 \;\; 1] L q$$

$$\sigma = \frac{E_e}{l_e} [-1 \;\; 1] \begin{bmatrix} l & m & 0 & 0 \\ 0 & 0 & l & m \end{bmatrix} q$$

$$\therefore \sigma = \frac{E_e}{l_e} [-l \; -m \;\; l \;\; m] q$$

Also, we can write 'σ' as,

$$\sigma = \frac{E_e}{l_e} [-l \; -m \;\; l \;\; m] \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{2x} \\ q_{2y} \end{bmatrix} \qquad \text{Since, } q = \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{2x} \\ q_{2y} \end{bmatrix}$$

Que. Determine the element stiffness matrix and element load vector for the plane truss element if it is subjected to tractional force of intensity P_o over its length.
Also explain how you will obtain stress and strain in the element from the element displacement vector. **[S-02, 13m]**

Problems

1) Consider the 3 bar truss shown in figure. Take $E = 200 \times 10^9$ N/m^2 and area of each member as 250 mm^2. Force P=25 KN is acting as shown in figure.
Determine:
i) Displacement at nodes ii) Stresses in each element iii) Reactions at supports **[S-09]**

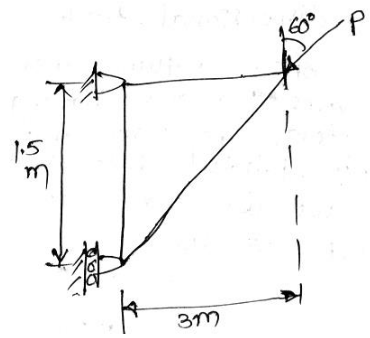

Fig.:

2) For the pin jointed configuration as shown in figure subjected to load P=50 KN, determine:
i) Nodal displacement ii) Stress in each element iii) Reaction at fixed support
Take Young's modulus, $E = 210\ GPa$ **[S-10, 14m]**

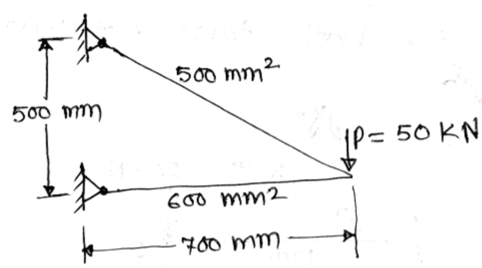

Fig.

3) For the plane truss shown in figure, determine the horizontal and vertical displacement of node 1 and the stresses in each element. All elements have $E = 210\ GPa$ and $A = 4.0 \times 10^{-4}\ m^2$. **[W-09, 13]**

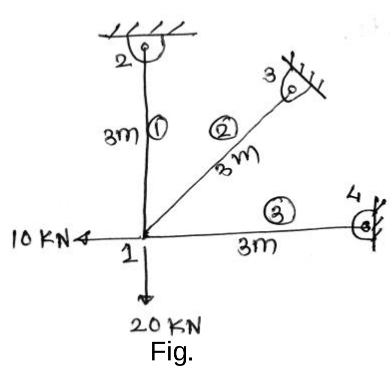

Fig.

4) For the plane truss supported by the spring at node 1 in figure, determine the nodal displacements and stresses in each element.
Let $E = 210\ GPa$ and $A = 5.0 \times 10^{-4}\ m^2$ for both truss elements. **[W-08]**

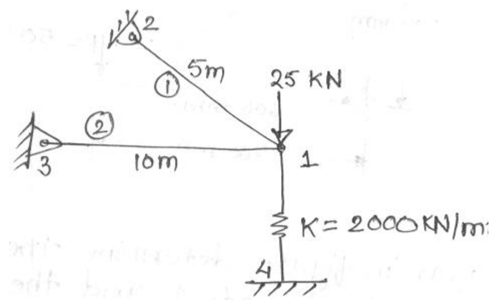

Fig.

5) Figure shows a truss consisting of three elements, cross-sectional area of each bar equals to 200 mm^2 and modulus of elasticity, $E = 210\ GPa$. Calculate deflection of nodes and stresses in each element. Also determine reactions at supports. **[W-07, 13m]**

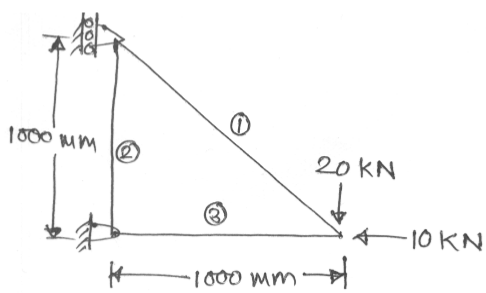

Fig.

6) Figure shows a two-dimensional simple truss with two members 1 and 2 having circular cross-section of diameters, d_1=30 mm and d_2=50 mm respectively. If the truss is pin jointed, determine the nodal displacement. The members are subjected to load P=100 N. Take E=200 GPa. **[S-07, 10m]**

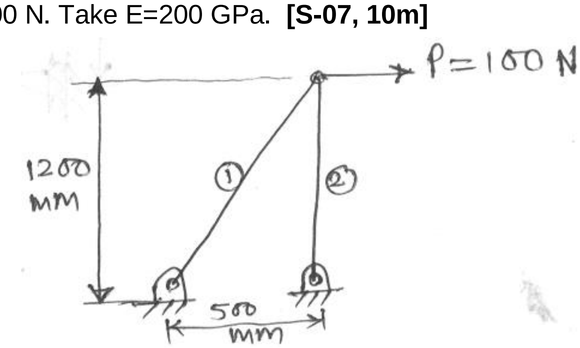

Fig.

7) A truss is shown in the figure. For that truss find the nodal displacements and element stresses.
A=200 mm^2 **[W-06, 13m]**
E=200 GPa.

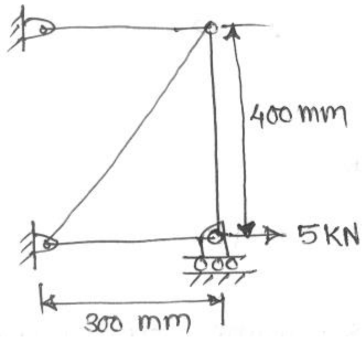

Fig.

8) The truss is shown in figure. Determine stresses in each member and reactions at support. Area of cross-section of each member is 200 mm^2 and $E = 200 \times 10^3\, MPa$. **[W-05, 14m]**

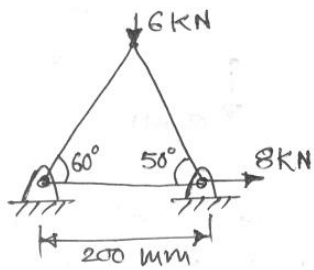

Fig.

9) A truss as shown in figure is subjected to the load of 3 KN. Determine the stresses in truss members. **[S-05]**

A=150*150 mm^2
$E = 120 \times 10^3\, N/mm^2$.

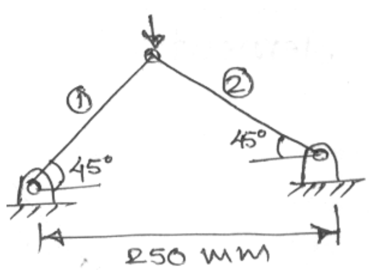

Fig.

10) For the truss shown in figure, find the displacement of each node and reaction at supports. E= 200 GPa and area of each element, A=200 mm^2. **[W-04]**

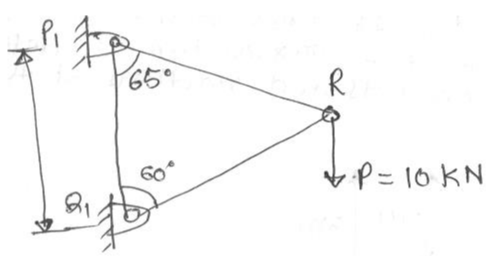

Fig.

11) Determine the stresses in the truss members as shown in figure. **[S-04]**
A=80 mm^2
$E = 210 \times 10^3 N/mm^2$.

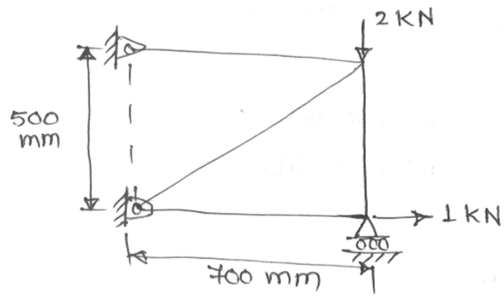

Fig.

12) A truss is shown in figure. Determine stresses and strains in each member. **[W-03, 13m]**
A=200 mm^2
$E = 200\ GPa$

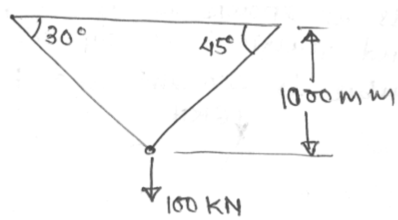

Fig.

13) A truss is shown in figure is subjected to the load of 7 KN. Determine deflection and stresses in the elements. **[S-03, 15m]**
A=400 mm^2
$E = 200 \times 10^3 N/mm^2$.

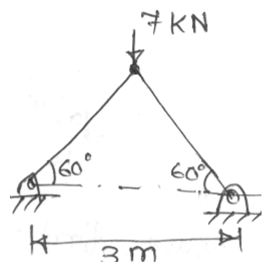

Fig.

14) A 2-D truss as shown in figure is subjected to loads F_1=20 KN and F_2= 5 KN. If cross-section area, A=300 mm^2 and modulus of elasticity, $E = 200 \times 10^3 N/mm^2$, determine the stresses in the truss elements and reactions at the supports. **[W-02, 16m]**

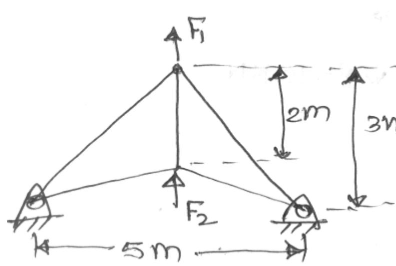

Fig.

15) A simple plane truss as shown in figure is subjected to load P=5000N. Determine the stress in the elements and reactions at the supports. **[W-01, 16m]**
Assume,
A = 300 mm^2
$E = 20 \times 10^4 N/mm^2$

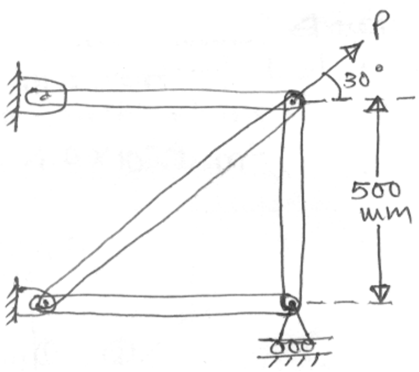

Fig.

16) A simple plane truss as shown in figure is made of two identical bars, and loaded as shown in the figure. Determine displacement and stresses. Cross-section area of each element is 100 mm^2.
$E = 200 \times 10^3 N/mm^2$.
Length of each bar = 150 mm
P_1=5 KN & P_2=3 KN **[S-01, 10m]**

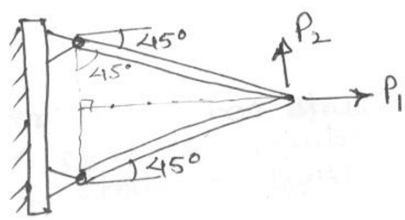

Fig.