KDK COLLEGE OF ENGINEERING, NAGPUR

Control system-II

Lecture Notes | Subject Code: BEELE701T

For 7th Sem. Electrical Engineering Student

DEPARTMENT OF ELECTRICAL ENGINEERING

BEELE701T – CONTROL SYSTEMS -II

Learning Objectives	Learning Outcomes
To impart knowledge of classical controller/compensator design for linear systems. To understand the theory and analyze non-linear system. To have idea about optimal and discrete time control system.	Students will be able to • Analyze the practical system for the desired specifications through classical and state variable approach. • Design the optimal control with and without constraints • Analyze non-linear and work with digital system and their further research.

UNIT - I

COMPENSATION: Need for compensation. Performance Analysis of Lead, Lag and Lag-lead Compensators in time & frequency domain, Bode Plots of Lead, Lag and Lag-lead Compensators. (Design of Compensator is not required).

UNIT-II

Solution of state equation: Review of state variable representations , diagonalization of state model ,eigen values and eigen vectors , generalized eigen vector, properties of state transition matrix (STM) , Computation of STM by Laplace transform, Cayley Hamilton theorem and Canonical transformation method. Solution of state equation.

UNIT-III

Design by state variable feedback: Controllability & observability. Kalman's test and Gilbert's test, duality, Design of State variable feedback. Effect of state feedback on controllability and observability.

UNIT-IV

Optimal Control System: Performance Index. Desirability of single P.I. Integral Square Error (ISE), Parseval's Theorem, parameter Optimization with & without constraints. Optimal control problem with T.F. approach for continuous time system only.

UNIT - V

Non Linear Control Systems: Types of non - linearities. jump resonance. Describing function analysis and its assumptions. Describing function of some common non-linearities. Singular points. Stability from nature of singular points. Limit cycles. Isocline method, Delta method.(Construction of phase trajectories is not expected)

UN1T-VI

Sampled Data Control Systems: Representation SDCS. Sampler & Hold circuit. Shanon's Sampling theorem, Z- Transform. Inverse Z- Transform & solution of Differencial Equations. 'Z' & 'S' domain relationship. Stability by Bi- linear transformation & Jury's test. Controllability &. observability of Discrete time systems.

BOOKS :

Text Books

Title of Book	Name of Author/s	Edition & Publisher
Control System Analysis	Nagrath & Gopal	New Age International
Linear Control System Analysis and Design	Constantine H. Houpis, Stuart N. Sheldon, John J. D'Azzo, Constantine H. Houpis, Stuart N. Sheldon	CRC Press
Digital Control and state variable methods	M. Gopal	The McGraw-Hill

Reference Books

Modern Control Engineering	k. Ogata	Prentice Hall
Modern control system	M.Gopal	New Age International
Modern Control Engineering	D.Roy Choudhury	PHI Learning Private Limited, New Delhi

UNIT - I

COMPENSATION

Compensator is an additional component or circuit that is inserted into a control system to equalize or compensate for a deficient performance.

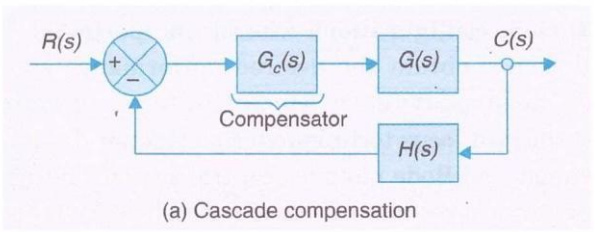
Necessities of compensation

1. In order to obtain the desired performance of the system, we use compensating networks. Compensating networks are applied to the system in the form of feed forward path gain adjustment.
2. Compensate a unstable system to make it stable.
3. A compensating network is used to minimize overshoot.
4. These compensating networks increase the steady state accuracy of the system. An important point to be noted here is that the increase in the steady state accuracy brings instability to the system.
5. Compensating networks also introduces poles and zeros in the system thereby causes changes in the transfer function of the system. Due to this, performance specifications of the system change.

Types of Compensator.

Series or Cascade compensation

Compensator can be inserted in the forward path as shown in fig below. The transfer function of compensator is denoted as $G_c(s)$, whereas that of the original process of the plant is denoted by G(s).



(a) Cascade compensation

Parallel or feedback compensation

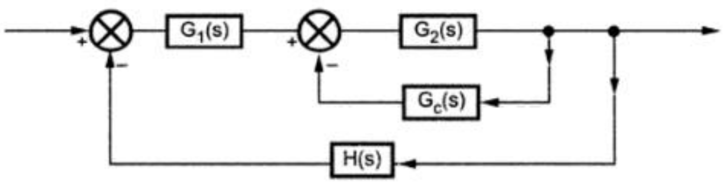


Fig: Parallel Compensator

The feedback is taken from some internal element and compensator is introduced in such a feedback path to provide an additional internal feedback loop. Such compensation is called feedback compensation or parallel compensation. The arrangement is shown in fig.

<u>Combined Cascade & feedback compensation or Series parallel compensator</u>

In some cases, it is necessary to provide both types of compensations, series as well as feedback. Such a scheme is called series – parallel compensation. The arrangement is shown in fig. below.

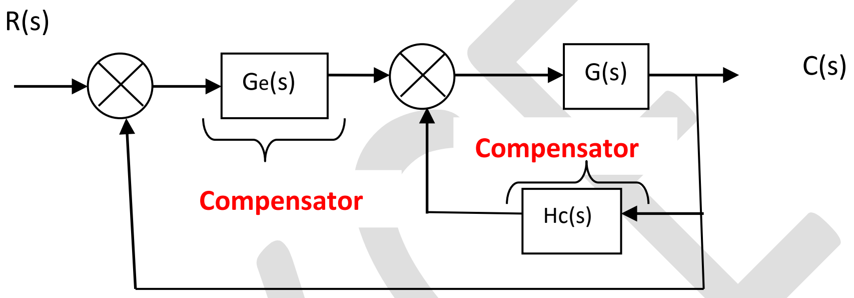


Fig: Series-parallel compensator

Compensator can be electrical, mechanical, pneumatic or hydrolic type of device. Mostly electrical networks are used as compensator in most of the control system. The very simplest of these are Lead, lag & lead-lag networks.

<u>what are compensating networks?</u>

A compensating network is one which makes some adjustments in order to make up for deficiencies in the system. Compensating devices are may be in the form of electrical, mechanical, hydraulic etc. Most electrical compensator is RC filter. The simplest networks used for electrical compensator are

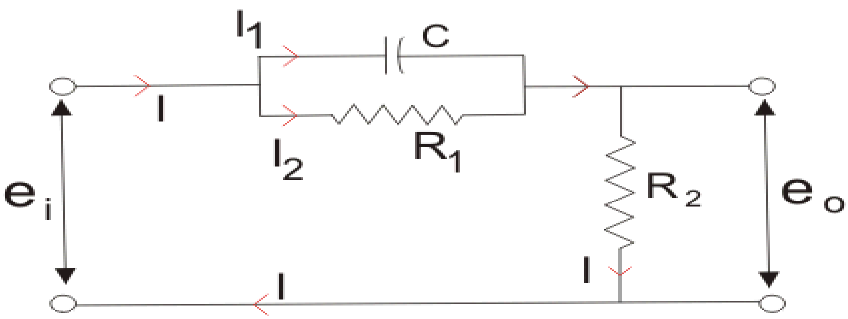
Lead compensator – (to speed up transient response, margin of stability and improve error constant in a limited way)

Lag compensator – (to improve error constant or steady-state behavior – while retaining transient response)

Lead – Lag compensator – (A combination of the above two i.e. to improve steady state as well as transient).

Phase Lead Compensation

A system which has one pole and one dominating zero (the zero which is closer to the origin than all over zeros is known as dominating zero.) is known as lead network. If we want to add a dominating zero for compensation in control system then we have to select lead compensation network. The basic requirement of the phase lead network is that all poles and zeros of the transfer function of the network must lie on (-)ve real axis interlacing each other with a zero located at the origin of nearest origin. Given below is the circuit diagram for the phase lead compensation network.



Phase Lead Compensation Network

Transfer Function = $G_c(s) = \frac{e_0(s)}{e_i(s)}$

$$\frac{e_0(s)}{e_i(s)} = \frac{R_2}{R_2 + \frac{R_1 x \frac{1}{Cs}}{R_1 + \frac{1}{Cs}}}$$

$$= \frac{R_2}{R_2 + \frac{R_1}{R_1 cs + 1}}$$

$$\frac{e_0(s)}{e_i(s)} = \frac{R_2(R_1Cs + 1)}{R_1 + R_2(R_1Cs + 1)}$$

$$\frac{e_0(s)}{e_i(s)} = \frac{R_2(R_1Cs + 1)}{R_1R_2Cs + R_1 + R_2}$$

$$= \frac{R_2}{R_1 + R_2}\left[\frac{R_1Cs + 1}{1 + \frac{R_2}{R_1 + R_2}R_1Cs}\right]$$

$$= \alpha\left[\frac{1 + Ts}{1 + \alpha Ts}\right] - - - - - - - - - - - - 1$$

Where $\alpha = \frac{R_2}{R_1 + R_2} < 1$

$$T = R_1C$$

Equation 1 can be written in the form of

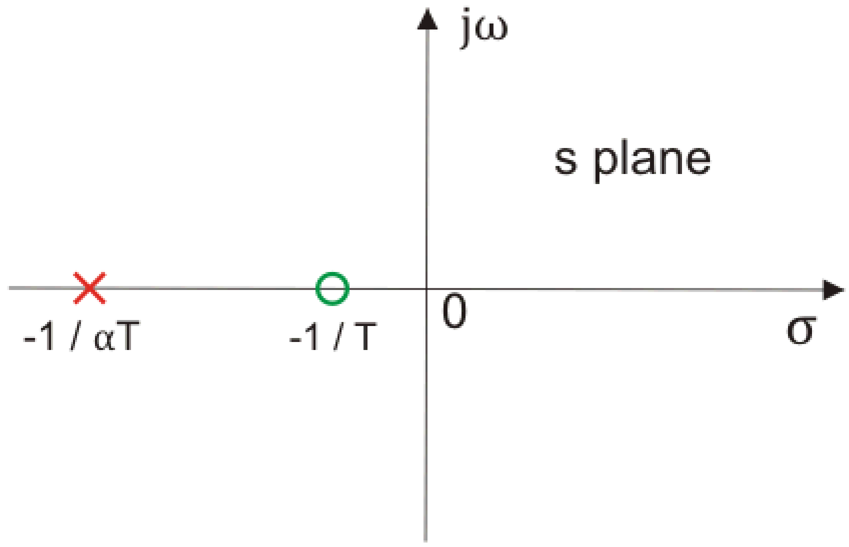
$$G_c(s) = \frac{\alpha T(s + \frac{1}{T})}{\alpha T(s + \frac{1}{\alpha T})}$$

$$G_c(s) = \frac{(s + \frac{1}{T})}{s + \frac{1}{\alpha T})}$$

$$= \frac{S + Z_c}{S + P_c}$$

Where $Z_c = \frac{1}{T}$ *and* $P_c = \frac{1}{\alpha T}$

Let us draw the pole zero plot for the above transfer function.



Pole Zero Plot of Lead Compensating Network

Zero is closer to origin so phase lead component is zero dominant.

The sinusoidal transfer function of the lead network is obtained by substituting S=jω in equation 1

$$G_c(j\omega) = \frac{e_0(j\omega)}{e_i(j\omega)} = \frac{\alpha(1 + j\omega T)}{(1 + j\omega\alpha T)}$$

Let $\lfloor G_c(j\omega) = \emptyset = \lfloor \frac{e_0(j\omega)}{e_i(j\omega)}$

$$\emptyset = \lfloor G_c(j\omega) = \tan^{-1}\omega T - \tan^{-1}\omega\alpha T \quad ----2$$

as α<1 we have ,

$$\tan^{-1}\omega\alpha T < \tan^{-1}\omega T$$

Φ is always positive

Therefore the output voltage always lead the input voltage in above network. Hence the above network is called lead network.

From equation 2 it is clear that for a given lead network, Φ is function of frequency . taking tan on both side of equation 2

$$\tan\emptyset = \frac{\omega T - \omega\alpha T}{1+\omega^2\alpha T^2} ----3$$

$$\varphi = \tan\emptyset = \frac{\omega T - \omega\alpha T}{1+\omega^2\alpha T^2}$$

When $\frac{d\varphi}{d\omega} = 0$ then φ is Maximum.

$$\frac{d\varphi}{d\omega} = \frac{(T-\alpha T)(1+\omega^2\alpha T^2) - (\omega T - \omega\alpha T)(2\omega\alpha T^2)}{1+\omega^2\alpha T^2} = 0$$

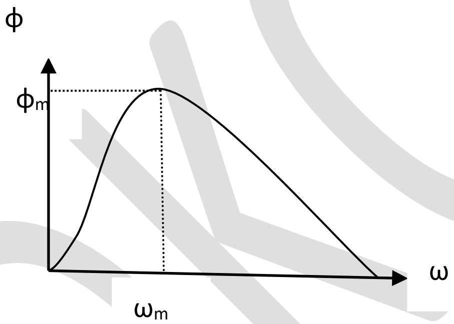
$$(T-\alpha T)(1+\omega^2\alpha T^2) - (\omega T - \omega\alpha T)(2\omega\alpha T^2) = 0$$

$$(T-\alpha T)[(1+\omega^2\alpha T^2) - 2\omega^2\alpha T^2] = 0$$

$$(1+\omega^2\alpha T^2) - 2\omega^2\alpha T^2 = 0$$

$$\omega^2\alpha T^2 = 1$$

$$\omega^2 = \frac{1}{\alpha T^2}$$



Variation of phase angle Φ as a function of ω .

$$\omega = \omega_m = \frac{1}{T\sqrt{\alpha}}$$

ω_m is the frequency at which maximum phase lead occurs, To find maximum phase lead Φ_m substitute

$\omega = \omega_m = \frac{1}{T\sqrt{\alpha}}$ in equation 3

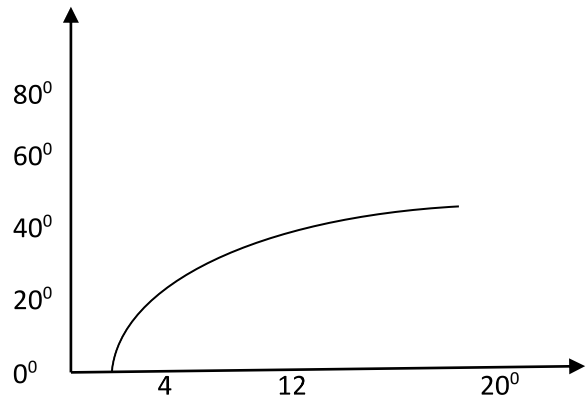
$$\tan\emptyset_m = \frac{\omega_m T - \omega_m\alpha T}{1+{\omega_m}^2\alpha T^2}$$

$$= \frac{\frac{1}{\sqrt{\alpha}} - \sqrt{\alpha}}{2} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\tan\emptyset_m = \frac{1-\alpha}{2\sqrt{\alpha}} --------------5$$

$$\sin\emptyset_m = \frac{1-\alpha}{1+\alpha} \ or \ \alpha = \frac{1-\sin\emptyset_m}{1+\sin\emptyset_m} ---6$$

Equation 6 is useful in computing α parameter of the network from the required maximum phase lead.



From above plot it is observed that

To obtain phase leads of more than 60^0 (i.e. α=0.8), $\frac{1}{\alpha}$ increases rather sharply

Hence to get phase lead > 60^0, two cascaded lead networks with moderate values of α rather than a single lead network with too small values of α

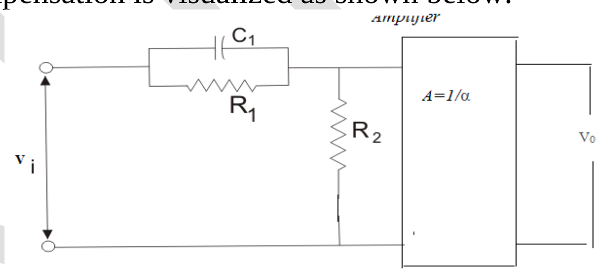
BODE PLOT FOR LEAD NETWORK

Consider a sinusoidal transfer function of the lead network

$$G_c(j\omega) = \frac{\alpha(1+j\omega T)}{(1+j\omega\alpha T)}, \quad \alpha < 1\text{-----}1$$

When ω=0, the network has a gain of α<1 or attenuation of 1/α in frequency domain compensation techniques, it is convenient to cancel the DC attenuation of the network with an amplification A=1/α.

Therefore the lead compensation is visualized as shown below.



Therefore sinusoidal transfer function of the lead compensator is

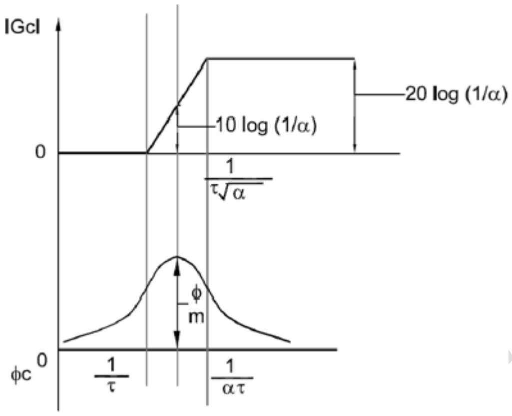
$$G_c(j\omega) = \frac{(1+j\omega T)}{(1+j\omega\alpha T)}, \quad \alpha < 1 \text{ ---------}2$$

LOG MAGNITUDE PLOT

Sr. No.	Factor	Corner Frequency	Description
1	1+jωT	$\omega_1 = \frac{1}{T}$	0db line upto corner frequency ω_1 and a line of slope equal to 20db/decade
2	$\frac{1}{1+j\omega\alpha T}$	$\omega_2 = \frac{1}{\alpha T}$	0db line upto corner frequency ω_2 and a line of slope equal to -20db/decade

Phase angle plot

$$\emptyset = \lfloor G_c(j\omega) = \tan^{-1}\omega T - \tan^{-1}\omega\alpha T$$



Lead Compensator Frequency Response

$$\omega_1\omega_2 = \frac{1}{T}\frac{1}{\alpha T} = \frac{1}{\alpha T^2} = \left(\frac{1}{T\sqrt{\alpha}}\right)^2$$

$$\omega_1\omega_2 = {\omega_m}^2 \; where \;\; \omega_m = \frac{1}{T\sqrt{\alpha}}$$

$$\omega_m = \sqrt{\omega_1\omega_2}$$

Where ω_m = frequency at which maximum phase lead occurs is the geometric means of the two corner frequencies $\omega_1 and \; \omega_2$

ω_1 = Lower Corner frequency

ω_2 = Upper Corner frequency

From the log magnitude plot

$$20 = \frac{Y_2 - 0}{\log_{10}\omega_2 - \log_{10}\omega_1}$$

$$Hence \;\; Y_2 = 20\log_{10}\frac{\omega_2}{\omega_1}$$

We have $\omega_1 = \frac{1}{T} \; and \; \omega_2 = \frac{1}{\alpha T}$

Therefore ,

$$Y_2 = 20\log_{10}\frac{1}{\alpha}$$

<u>Effect of Phase Lead Compensation</u>

1. The velocity constant K_v increases.
2. The slope of the magnitude plot reduces at the gain crossover frequency so that relative stability improves and error decrease due to error is directly proportional to the slope.
3. Phase margin increases.
4. Response becomes faster.

Advantages of Phase Lead Compensation

Let us discuss some of the advantages of the phase lead compensation-

1. Due to the presence of phase lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
2. Due to the presence of phase lead compensation maximum overshoot of the system decreases.

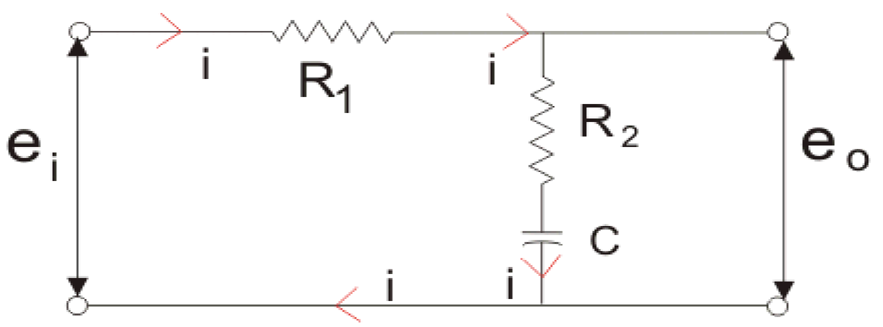
Disadvantages of Phase Lead Compensation

Some of the disadvantages of the phase lead compensation -

1. Steady state error is not improved.

Lag Network

A system which has one zero and one dominating pole (the pole which is closer to origin that all other poles is known as dominating pole) is known as lag network. If we want to add a dominating pole for **compensation in control system** then, we have to select a **lag compensation** network. The basic requirement of the phase lag network is that all poles and zeros of the transfer function of the network must lie in (-)ve real axis interlacing each other with a pole located or on the nearest to the origin. Given below is the circuit diagram for the phase **lag compensation** network



Phase Lag Compensating Network

Transfer Function

$$\text{Transfer Function} = G_c(s) = \frac{e_0(s)}{e_i(s)}$$

$$\frac{e_0(s)}{e_i(s)} = \frac{\left[R_2 + \frac{1}{Cs}\right] I_{(s)}}{\left[R_1 + R_2 + \frac{1}{Cs}\right] I_{(s)}}$$

$$= \frac{R_2Cs + 1}{1 + (R_1 + R_2)Cs}$$

$$\frac{R_2Cs + 1}{1 + \left(\frac{R_1 + R_2}{R_2}\right) R_2Cs}$$

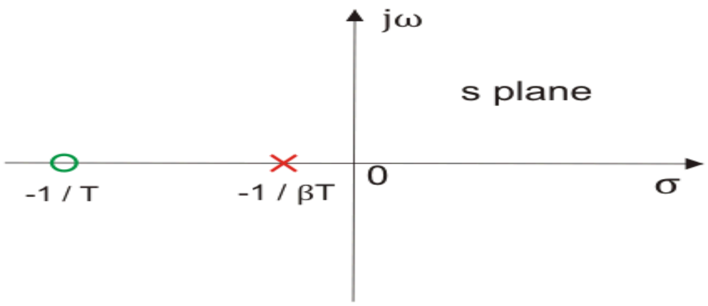
Let $\tau = R_2C$ and $\beta = \frac{R_1+R_2}{R_2} > 1$

$$G_c(s) = \frac{1+s\tau}{1+\beta s\tau} ---------1$$

$$G_c(s) = \frac{e_0(s)}{e_i(s)} = \frac{1}{\beta}\left[\frac{s+\frac{1}{\tau}}{s+\frac{1}{\beta\tau}}\right] = \frac{1}{\beta}\left[\frac{s+Z_c}{s+P_c}\right]$$

Where $Z_c = \frac{1}{\tau}$ and $P_c = \frac{1}{\tau\beta}$

The pole zero location of the lag network is as shown in figure below.



Pole Zero Plot of Lag Network

To obtain sinusoidal transfer function we put s=jω in the equation 1

$$G_c(\text{j}\omega) = \frac{e_0(\text{j}\omega)}{e_i(\text{j}\omega)} = \frac{1+\text{j}\omega\tau}{1+\text{j}\omega\beta\tau}$$

If $\emptyset_{m=\angle G(j\omega)}$then

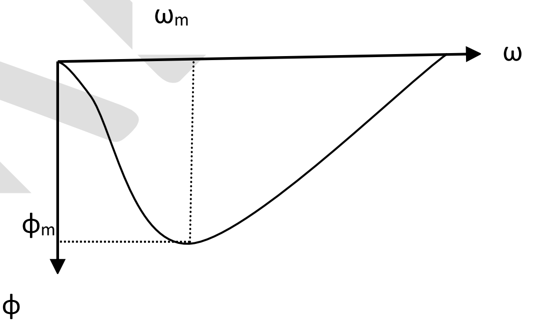
$$\emptyset = \angle G_c(\text{j}\omega) = \tan^{-1}\omega\tau - \tan^{-1}\omega\beta\tau ----2$$

Since β>1, $\tan^{-1}\omega\beta\tau > \tan^{-1}\omega\tau$

Or φ_m is negative

Therefore the output voltage lags the input voltage. Hence the name lag Network.

The phase lag characteristics as a function of frequency ω is as shown below.



Taking tan on both sides of equation 2

$$\tan\emptyset = \frac{\omega\tau - \omega\beta\tau}{1-\omega^2\beta\tau^2}$$

Let ψ= tan φ

Therefore $\varphi = \frac{\omega\tau-\omega\beta\tau}{1-\omega^2\beta\tau^2}$

For the phase angle φ to be maximum tan φ= ψ should be maximum

i.e. $\frac{d\varphi}{d\omega} = 0$

i.e. $\frac{d\varphi}{d\omega} = \frac{(1+\omega^2\beta\tau^2)(\tau-\beta\tau)-(\omega\tau-\omega\beta\tau)(2\omega\beta\tau^2)}{(1+\omega^2\beta\tau^2)} = 0$

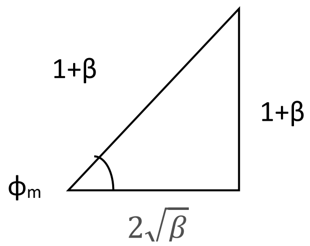
after solving, we get

$$\omega = \omega_m = \frac{1}{\tau\sqrt{\beta}}$$

Frequency at which maximum phase lag φ_m occurs is

$\omega_m = \frac{1}{\tau\sqrt{\beta}}$

i.e. $\tan\varphi_m = \frac{1-\beta}{2\sqrt{\beta}}$



therefore $\sin\varphi_m = \frac{1-\beta}{1+\beta}$

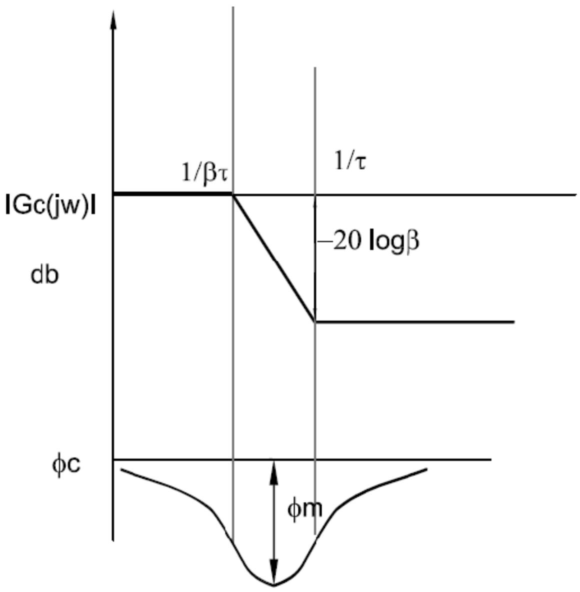
or $\beta = \frac{1-\sin\phi_m}{1+\sin\phi_m}$

BODE PLOT OF LAG NETWORK

Sr. No.	Factor	Corner Frequency	Description
1	$\frac{1}{1+j\omega\beta\tau}$	$\omega_1 = \frac{1}{\beta\tau}$	0db line upto corner frequency ω_1 and a line of slope equal to -20db/decade
2	$1+j\omega\tau$	$\omega_2 = \frac{1}{\tau}$	0db line upto corner frequency ω_2 and a line of slope equal to 20db/decade

Phase angle Plot

$$\phi = \lfloor G_c(j\omega) = \tan^{-1}\omega\tau - \tan^{-1}\omega\beta\tau$$



Bode Plot for lag Compensator

$$\omega_1\omega_2 = \frac{1}{\beta\tau^2} = \frac{1}{\left(\tau\sqrt{\beta}\right)^2} = \omega_m^2$$

$$\omega_m = \sqrt{\omega_1\omega_2}$$

The frequency at which maximum phase lag occurs is the geometrical mean of the two corner frequencies ω_1 and ω_2

$$-20 = \frac{Y_2 - 0}{\log_{10}\omega_2 - \log_{10}\omega_1}$$

$$Y_2 = 20\log\frac{\omega_1}{\omega_2}$$

i.e. $Y_2 = 20\log_{10}\left[\frac{1/\beta\tau}{1/\tau}\right]$

$$Y_2 = 20\log_{10}\left[\frac{1}{\beta}\right]$$

<u>Effect of Phase Lag Compensation</u>

1. Gain crossover frequency increases.
2. Bandwidth decreases.
3. Phase margin will be increase.
4. Response will be slower before due to decreasing bandwidth, the rise time and the settling time become larger.

<u>Advantages of Phase Lag Compensation</u>

Let us discuss some of the advantages of phase lag compensation -

1. Phase lag network allows low frequencies and high frequencies are attenuated.
2. Due to the presence of phase lag compensation the steady state accuracy increases.

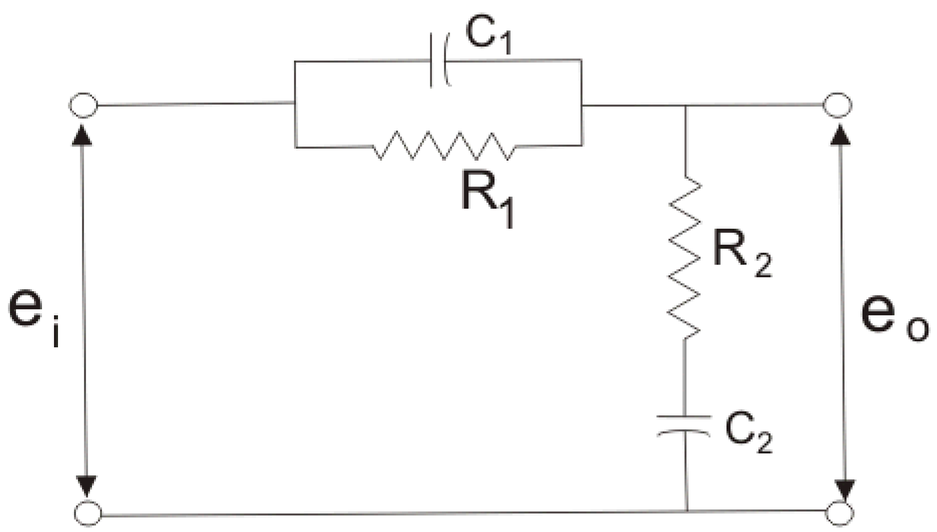
<u>Disadvantages of Phase Lag Compensation</u>

Some of the disadvantages of the phase lag compensation -

1. Due to the presence of phase lag compensation the speed of the system decreases.

Phase Lag Lead Network

With single lag or lead compensation may not satisfied design specifications. For an unstable uncompensated system, lead compensation provides fast response but does not provide enough phase margin whereas lag compensation stabilize the system but does not provide enough bandwidth. So we need multiple compensators in cascade. Given below is the circuit diagram for the phase **lag- lead compensation** network.



Lag Lead Compensating Network

Now let us determine transfer function for the given network and the transfer function can be determined by finding the ratio of the output voltage to the input voltage.

$$G_c(s) = \frac{\left(s + \frac{1}{\tau_1}\right)}{\left(s + \frac{1}{\beta\tau_1}\right)} \frac{\left(s + \frac{1}{\tau_2}\right)}{\left(s + \frac{1}{\alpha\tau_2}\right)} \quad \alpha < 1, \beta > 1$$

$$G_c(s) = \frac{(1 + s\tau_1)(1 + s\tau_2)/\tau_1\tau_2}{s^2 + s\left(\frac{1}{\beta\tau_1} + \frac{1}{\alpha\tau_2}\right) + \frac{1}{\alpha\beta\tau_1\tau_2}}$$

$$= \frac{(1+s\tau_1)(1+s\tau_2)}{\tau_1\tau_2 s^2 + s\left(\frac{\tau_1}{\alpha}+\frac{\tau_2}{\beta}\right)+\frac{1}{\alpha\beta}} ------------1$$

We have,

$$e_0(s) = \left[R_2 + \frac{1}{C_2 s}\right] I(s)$$

$$e_i(s) = \left[\frac{R_1 \frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} + R_2 + \frac{1}{C_2 s}\right] I(s)$$

$$\frac{e_i(s)}{\beta e_o(s)} = \frac{\left[\frac{R_1}{R_1 C_1 s + 1} + R_2 + \frac{1}{C_2 s}\right]}{\left[R_2 + \frac{1}{C_2 s}\right]}$$

$$\frac{e_i(s)}{e_o(s)} = \frac{R_1 C_1 s + (R_2 C_2 s + 1)(R_1 C_1 s + 1)}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

$$G_c(s) = \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_1 R_2 C_1 C_2 s^2 + s(R_1 C_1 + R_2 C_2 + R_1 C_2) + 1} -----2$$

Comparing equation 1 and 2

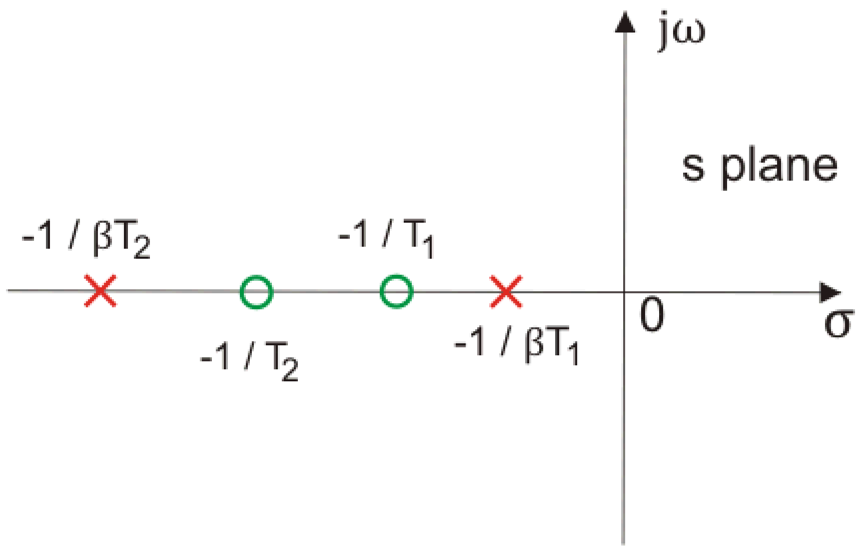
$$\tau_1 = R_1 C_1 \; and \; \tau_2 = R_2 C_2$$

$$\frac{\tau_1}{\alpha} + \frac{\tau_2}{\beta} = R_1 C_1 + R_2 C_2 + R_1 C_2$$

$$\frac{1}{\alpha\beta} = 1 \; therefore \;\; \alpha\beta = 1$$

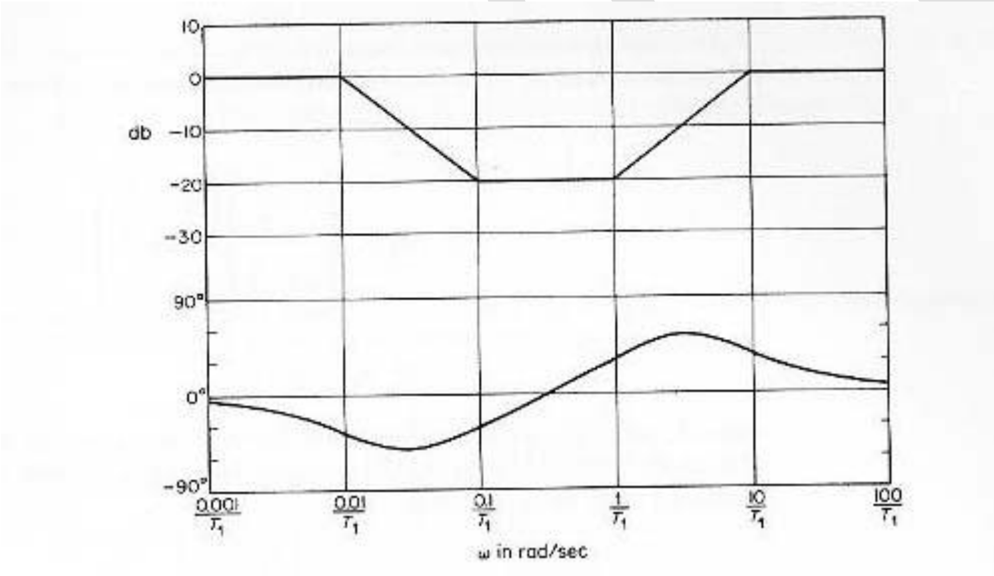
A single lag- lead network doesnot permit an independent choice of α and β

$$G_c(s) = \frac{\left(s + \frac{1}{\tau_1}\right)}{\left(s + \frac{1}{\beta\tau_1}\right)} \cdot \frac{\left(s + \frac{1}{\tau_2}\right)}{\left(s + \frac{1}{\beta\tau_2}\right)} \quad means \; \frac{1}{\alpha} = \beta \; and \; \beta > 1$$



Pole Zero Plot Lag Lead Network

BODE PLOT OF LAG-LEAD NETWORK



Advantages of Phase Lag Lead Compensation

Let us discuss some of the advantages of phase lag- lead compensation-

1. Due to the presence of phase lag-lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
2. Due to the presence of phase lag-lead network accuracy is improved.

Comparison between Lead Compensator and Lag Compensator

LAG Network	LEAD Network
• Improve Steady-state response	• Improve Transient response
• Has low pass filter characteristic	• Has high pass filter characteristic
• Offers unity gain to the low frequency signals and a gain of $1/\beta << 1$ to high frequency signal	• Offers unity gain to the low frequency signals and a gain of $1/\alpha >> 1$ to high frequency signal
• Signal to noise ratio is better	• Signal to noise ratio is poorer
• The gain cross over frequency and the band width of the system is reduced	• The gain cross over frequency and the band width of the system is increase.
• Decreased bandwidth slows the response of the system	• Increased band width makes the response faster.
• It decreases the phase shift.	• Increases phase shift.
• Here pole is closer to origin than the zero.	• Here zero is closer to origin than the pole.