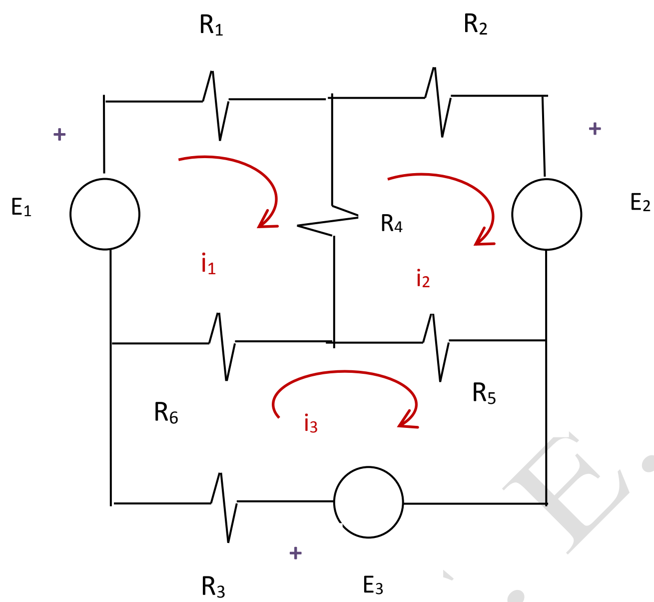
Mesh Analysis:

Mesh analysis is used to analyze any electrical network for its responses.

Consider the following network



The above network consists of three meshes (Smallest closed path similar to a pane of a window). It has been assumed that these meshes carry loop currents i_1, i_2, i_3 as shown in the figure above.

Writing KVL equations for the mesh containing loop current i_1, we get,

$R_1 i_1 + R_4 (i_1 - i_2) + R_6 (I_1 - i_3) = E_1$ … 1

Writing KVL equations for the mesh containing loop current i_2, we get,

$R_2 i_2 + R_5 (i_2 - i_3) + R_4 (I_2 - i_1) = E_2$ … 2

Writing KVL equations for the mesh containing loop current i_3, we get,

$R_3 i_3 + R_6(i_3 - i_1) + R_5(I_3 - i_2) = E_3$ … 3

Rearranging these three equations we get,

$(R_1 + R_4 + R_6)\, i_1 \; - R_4 I_2 - R_6 i_3 = E_1$ … 4

$- R_4 I_1 + (R_2 + R_4 + R_5)\, i_2 \; - R_5 i_3 = E_2$ … 5

$- R_6 I_1 - R_5 i_2 + (R_3 + R_6 + R_5)\, i_2 \; = E_3$ … 6

Equations 4, 5 and 6 can be put in matrix form as under

$$\begin{bmatrix} (R_1 + R_4 + R_6) & -R_4 & -R_6 \\ -R_4 & (R_2 + R_4 + R5) & -R_5 \\ -R_6 & -R_5 & (R_3 + R_6 + R5) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad \cdots \quad 7$$

Using crammer's rule solution for mesh currents i_1, i_2, i_3 can be obtained.

Equation 7, in general can be written as.

$$\begin{bmatrix} (Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad \cdots \quad 8$$

Where $Z_{11} = (R_1 + R_4 + R_6)$ = is the sum of the resistances (more generally the impedances) of a particular mesh which is called as the "self impedance" of that mesh.

$Z_{12} = Z_{21} = (-R_4)$ = is the negative of the sum of the impedances common between 1st and the 2nd mesh and is called the "mutual impedance" between first and the second mesh.

Applying crammers rule one can find the mesh currents of any mesh of interest.

Note: While applying mesh equations for analysis, it is preferable to have voltage sources in the network.

Example:

Find the current flowing through $1 - \Omega$ as shown in 'Fig. 1 (b)'.

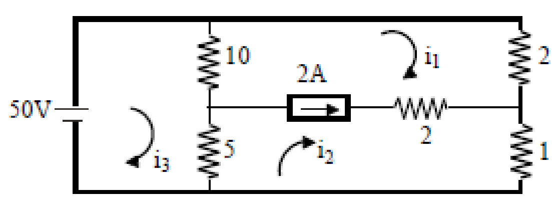


Fig. 1(b)

Solution: 2 Ω resistance in series with the current source of 2 A doesn't contribute to any change in current in various branches whether it is present or absent. Hence replacing it by a short circuit and writing KVL equations we get,

$15\,i_3 - 10\,i_1 - 5\,i_2 = 50 \quad \ldots \quad 1$

$-15\,i_3 + 12\,i_1 + 6\,i_2 = 0 \quad \ldots \quad 2$

$i_1 - i_2 = 2 \quad \ldots \quad 3$

$$\begin{bmatrix} (-10 & -5 & 15 \\ 12 & 6 & -15 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \\ 2 \end{bmatrix}; \qquad \Delta = \begin{vmatrix} -10 & -5 & 15 \\ 12 & 6 & -15 \\ 1 & -1 & 0 \end{vmatrix}$$

Applying crammer's rule we get,

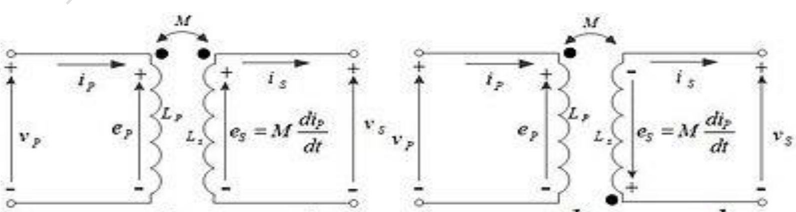
$$i_1 = \frac{\begin{vmatrix} 50 & -5 & 15 \\ 0 & 6 & -15 \\ -2 & -1 & 0 \end{vmatrix}}{\Delta}; \qquad i_2 = \frac{\begin{vmatrix} -10 & 50 & 15 \\ 12 & 0 & -15 \\ 1 & 2 & 0 \end{vmatrix}}{\Delta}; \qquad i_3 = \frac{\begin{vmatrix} -10 & -5 & 50 \\ 12 & 6 & 0 \\ 1 & -1 & 2 \end{vmatrix}}{\Delta}$$

Solving we get, i_1 = 16 A; i_2 = 18 A and i_3 = 20 A

Mesh analysis of networks with dependant sources:

Dot Convention:

In any electrical network magnetically coupled coils give rise to dependent sources. A current in one coil is capable of producing a mutually induced emf in a coupled coil. However the direction of emf so induced depends upon but the polarity of coupling and is indicated by dots at the coupled coils.



$$v_P = L_P\frac{di_P}{dt} + M\frac{di_s}{dt} \qquad v_P = L_P\frac{di_P}{dt} - M\frac{di_s}{dt}$$

$$v_S = M\frac{di_P}{dt} + L_S\frac{di_s}{dt} \qquad v_S = -M\frac{di_P}{dt} + L_S\frac{di_s}{dt}$$

The current entering the dot in a coil will induce a positive emf at the dotted end of the other coupled coil. On the other hand coil leaving the dot of a coil will produce an emf at the un-dotted end of the coupled coil. This is illustrated in the figure above.

Write the mesh equations in matrix form for the network shown in Fig 1. 'b'.

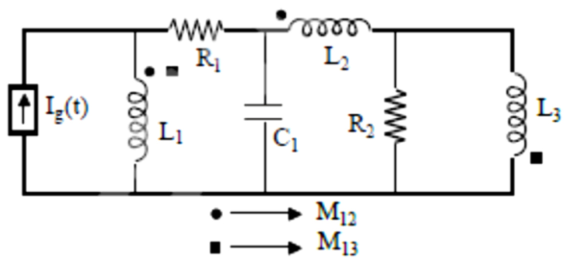
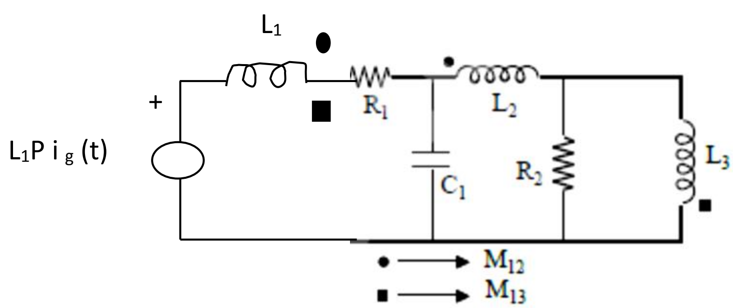


Fig. 1(b)



Assuming the three mesh currents as i_1, i_2 and i_3 in clockwise direction and writing mesh equations for the three meshes in matrix form, we get,

$$\begin{bmatrix} L_1P + R_1 + \dfrac{1}{C_1P} & -M_{12}P - \dfrac{1}{C_1P} & M_{13}P \\ -M_{12}P - \dfrac{1}{C_1P} & L_2P + R_2 + \dfrac{1}{C_1P} & -R_2 \\ M_{13}P & -R_2 & R_2 + L_3P \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} L_1P\, i_g(t) \\ 0 \\ 0 \end{bmatrix} \quad where P = \frac{d}{dt} \; and \; \frac{1}{P} = \int dt$$