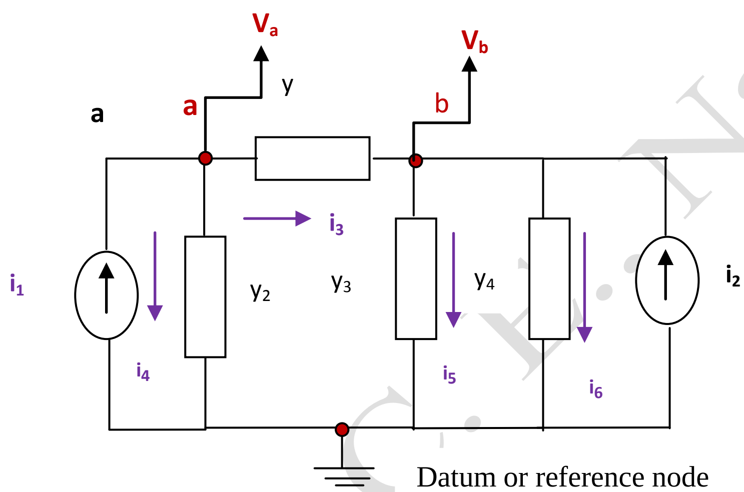
Unit II:

Content: Nodal basis equilibrium equations, matrix for electrical network containing independent sources and reactances, Duality.

NODAL ANALYSIS: Basis of nodal analysis is Kirchhoff's current law.

In this method of analysis, one of the nodes is considered to be the reference node and the potential of all other nodes are expressed with respect to the datum or reference node.

Consider the following network having three nodes.



At Node 'a', applying KCL we get,

$i_1 = i_3 + i_4$ … 2.1

but $i_3 = (V_a - V_b)y_1$ and $i_4 = V_a\, y_2$

Substituting for i_3 , i_4 and simplifying equation 2.1, we get,

$i_1 = (y_1 + y_2)\, V_a - y_1\, V_b$ … 2.2

Applying KCL at Node 'b' we get,

$i_2 + i_3 = i_5 + i_6$ … 2.3

$i_5 = V_b\, y_3$ and $i_6 = V_b y_4$

On substitution equation 2.3 becomes,

$i_2 + (V_a - V_b)y_1 = V_b\, y_3 + V_b y_4$

$i_2 = -y_1 V_a + (y_1 + y_3 + y_4) V_b$ … 2.4

Equations 2.2 and 2.4 can be written as,

$y_{11} V_a + y_{12} V_b = i_1$ … 2.5

$y_{21} V_a + y_{22} V_b = i_2$ … 2.6

y_{11} is called the self admittance at node 1 which includes the sum of all admittances at node 1

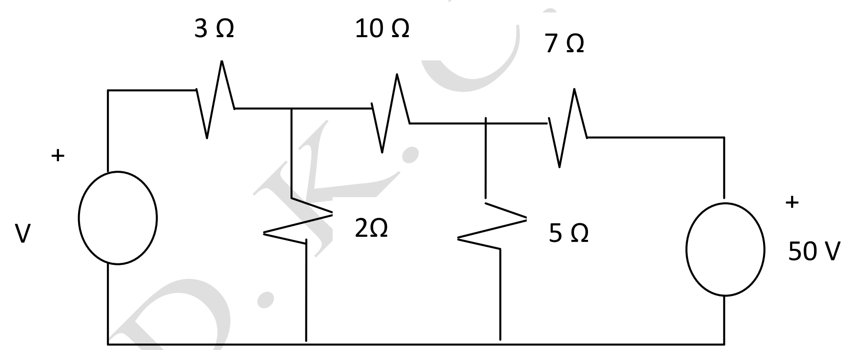
$y_{12} = y_{21}$ is called the mutual admittance between the node pairs 'a' and 'b' which is negative of the sum of admittances between the given node pair a, b.

Using crammer's rule one can solve for V_a and V_b.

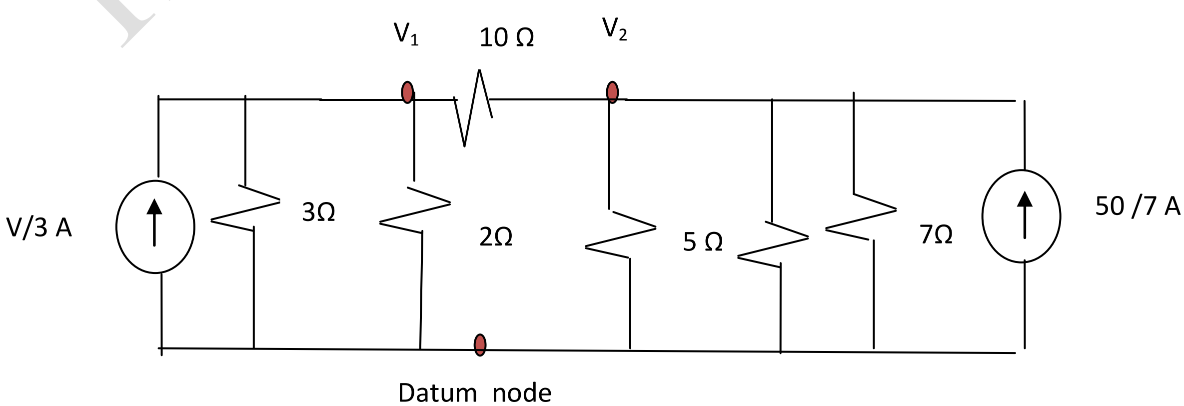
Note: It is preferable to convert voltage sources to their equivalent current sources in case the network has voltage sources. However it is not compulsory to do so. One can carry out the analysis with voltage sources also.

Example:

Find the voltage V in the circuit shown below which makes the current in 10 ohm resistor zero by nodal analysis.



Converting the voltage sources and rewriting the network we get,



Applying KCL at node 1,

V_1 (1/3 + 1/2 +1/10) –v_2 (1/10) = V/3

0.933 V_1 - 0.1 V_2 = 0.33 V … 1

Applying KCL at node 2,

- (1/10) V_1 +(1/10+1/5+1/7) V_2 = 7.1428

- 0.1 V_1 +0.4428 V_2 = 7.1428 … 2

From 1 and 2,

$$v_1 = \frac{\begin{vmatrix} 0.33v & -0.1 \\ 7.1428 & 0.4428 \end{vmatrix}}{\Delta} \qquad v_2 = \frac{\begin{vmatrix} 0.933 & 0.33v \\ -0.1 & 7.1428 \end{vmatrix}}{\Delta}$$

Given that the current through 10 ohm resistor is zero.

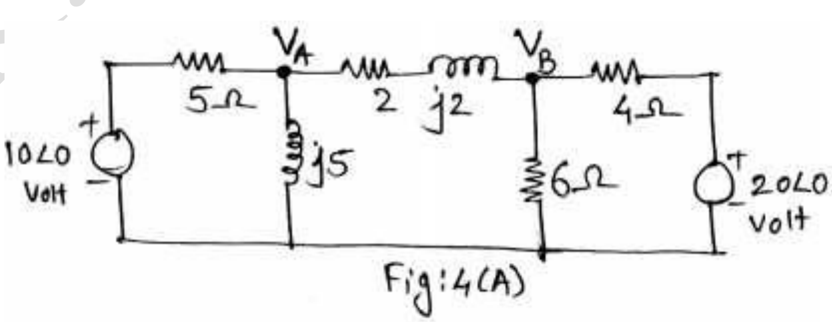
i.e. $I_{10\Omega} = \dfrac{v_1 - v_2}{10} = 0$

Substituting for v_1 and v_2 from above and solving, we get, **V= 52. 9 volts.**

Practice problems on Node Analysis”

RTMNU Winter 2017 Examination.

Unig nodal analysis find the current in the branch AB



Using nodal analysis fine Vb such that the current in (2 + j 3) Ω is zero

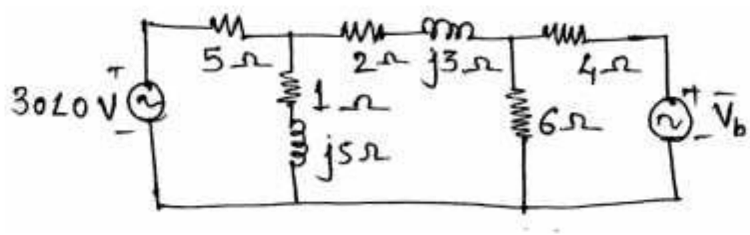


Fig.3.(A).

RTMNU Summer 2017 examination.

Write the nodal equations in matrix form as shown in 'Fig. 4 (a)'.

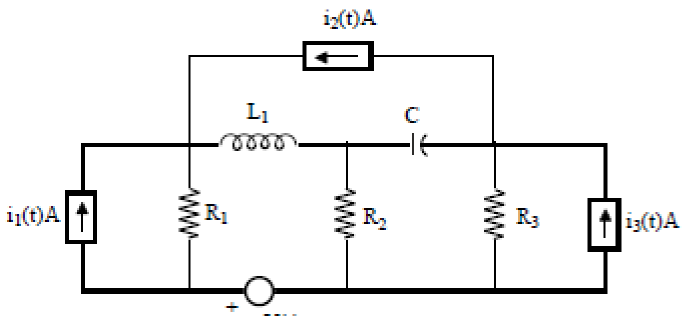


Fig. 4(a)

Find 'V_{ba}' for the network shown in 'Fig. 4 (b)' using NODAL ANALYSIS.

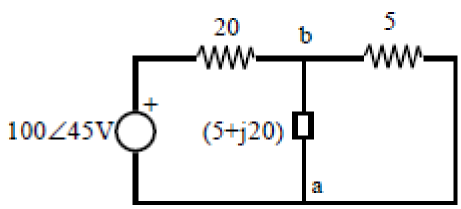
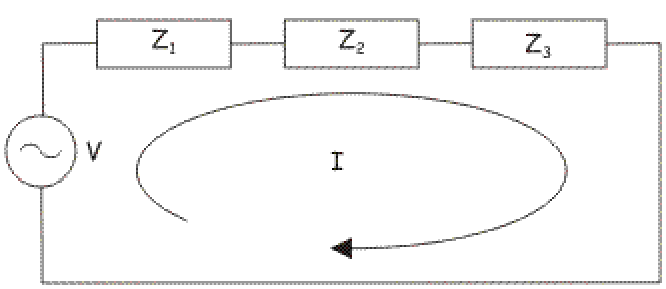


Fig. 4(b)

Dual Network

Two Electrical Network are said to be dual, if the mesh equations of one is the node equation of other.

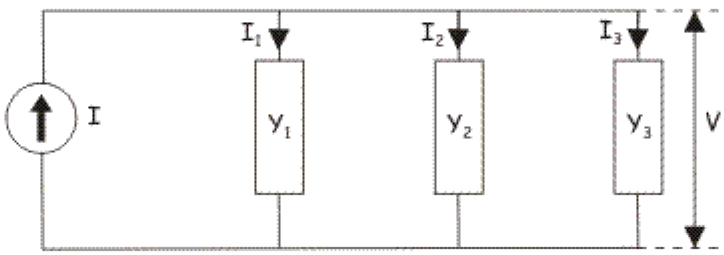
The **dual network** are based on Kirchhoff Current Law and Kirchhoff Voltage Law.



Network A

Applying Kirchhoff Voltage Law in the network A, above we get,

$$V = IZ_1 + IZ_2 + IZ_3$$
$$\Rightarrow V = I(Z_1 + Z_2 + Z_3) \quad \cdots\cdots \quad (i)$$



Network B

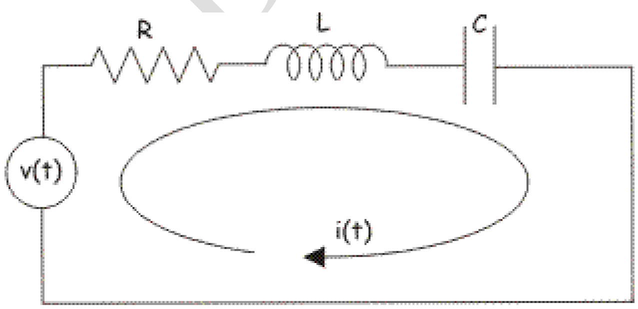
Applying Kirchhoff Current Law in the network B, above we get,

$$I = I_1 + I_2 + I_3$$
$$\Rightarrow I = VY_1 + VY_2 + VY_3$$
$$\Rightarrow I = V(Y_1 + Y_2 + Y_3) \quad \cdots\cdots \quad (ii)$$

Here we have found that equation (i) and (ii) are similar in their mathematical form. Equation (i) is in mesh form and equation (ii) is in nodal form. Here, left side variable of equation (i) is voltage and left side variable of equation (ii) is current. Similarly, right side of equation (i) is product of current and total impedance of the circuit. Similarly, right side of equation (ii) is the product of voltage and admittance of the circuit. So, it is needless to say these two networks are **dual network**. From, the examples it is also clear that **dual network** may not be equivalent network. The circuit equation of two dual network are similar in form but the variable are interchanged.

Construction Of Dual Network

Let us consider series RLC circuit as shown below.



Network C

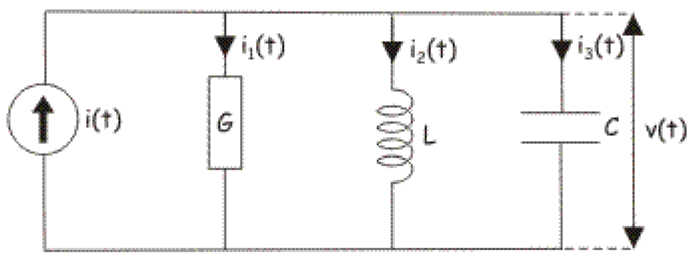
Applying Kirchhoff Voltage Law in this circuit, we get,

$$v(t) = Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i(t)dt \quad \cdots\cdots \quad (iii)$$

Let us replace all the variables and constants by their dual in the equation. By doing that, we get,

$$i(t) = Gv(t) + C\frac{\mathrm{d}v(t)}{\mathrm{d}t} + \frac{1}{L}\int v(t)dt \quad \cdots\cdots \quad (iv)$$

The electrical network drawn by the circuit equation (iv), would be



Network D

$$i_1(t) = Gv(t)$$

$$i_2(t) = C\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

$$i_3(t) = \frac{1}{L}\int v(t)dt$$

It is needless to say

$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

This is nothing but Kirchhoff Current Law. As per definition of dual network, the network C and network D are dual to each other.

Table For Dual Elements.

Original Element	Dual Element
Resistance(R)	Conductance(G)
Inductance(L)	Capacitance(c)
Series Branch	Parallel Branch
Switch open	Switch Closed
Charge	Flux Linkages
Mesh	Node