

# Unit-I

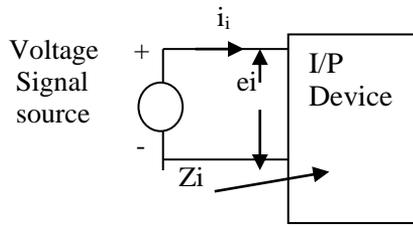
## Loading Effects:-

In a measurement system, when an element is introduced may be for the purpose of signal sensing, conditioning or detection the original signal should remain undistorted under ideal situation. However practically it is not possible. There is extraction of energy from the system which causes distortion. This distortion may be in the form of attenuation, waveform distortion, phase shift and may be all these features put together.

The incapability of the system to faithfully measure, record or control the input signal (measured) in undistorted form is called loading effect.

To study loading effects due to shunt connected and series connected instruments it is important to know about the input and output impedances. Since the loading effects are due to impedances of various elements connected in a system, so it is desirable to analyze their effects.

## Input impedance:-



A voltage signal source and input device connected across it is shown in fig. The magnitude of the input impedance is given by

$$Z_i = e_i / i_i$$

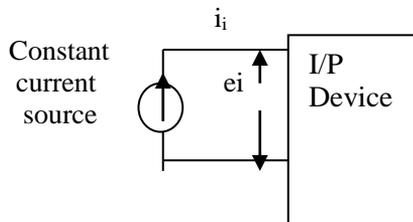
The instantaneous power extracted by the input device is,

$$P = e_i * i_i / Z_i$$

Thus when an instrument connected across the system for measurements, the system conditions change from the original on account of the power drawn from the system and the amount by which they deviate from the original conditions depend upon input impedance of the device. For lower values of input impedance, power extraction more and hence the power distortion of the original signal is high.

Ideally when a voltmeter is connected to the system should not draw any power from the system and this is possible only if voltmeter has an impedance that has infinite value.

Input admittance:-



A constant current source and input device connected across it is shown in fig.

The magnitude of the input impedance is given by,

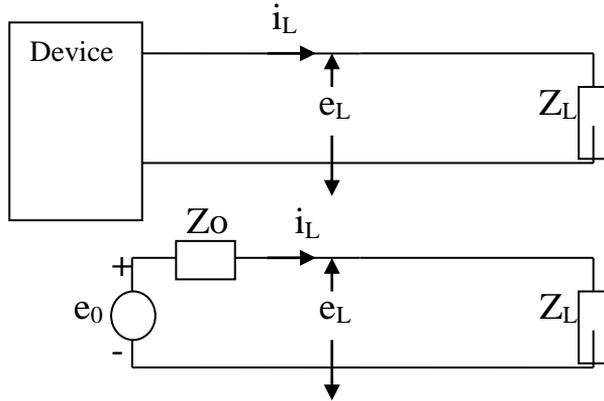
$$Y_i = i_i / e_i, \quad Z_i = e_i / i_i = 1 / Y_i$$

The instantaneous power extracted by the input device is,

$$P = i_i * e_i = i_i^2 / Y_i = i_i^2 * Z_i$$

From above it is clear that in case of series elements the power drawn from the current signal source is small when the input admittance of the device is high that is loading effects are small. An ammeter, which is a series device should be designed with a low input impedance so that the current is correctly measured. Ideally an ammeter should have an infinite input admittance.

### Output Impedance-



The output impedance is defined as its equivalent impedance as seen by the load. This definition is only meaningful for an active device at a pair of terminals considered as source. The equivalent impedance term implies that the device can be represented by a Thevenin's equivalent circuit.

Let,

$e_o$ - Voltage across o/p terminals of the device when the load is not connected

$e_L$ - Voltage across o/p terminals of the device when the load is connected

O/P impedance of active device is

$$Z_o = (e_o - e_L) / i_L$$

$$(e_o - e_L) = i_L * Z_o$$

$$\text{Power loss in voltage source, } p = (e_o - e_L) * i_L = i_L^2 * Z_o$$

Thus for voltage sources, the lower the output impedance lower is the voltage drop and lower is power consumption. Ideally there should not be any loading effect and this requires that output impedance  $Z_o$  of the voltage source be equal to zero.

### Output admittance:-

Output admittance is considered while dealing with current sources, which is defined as,

$$Y_o = (i_o - i_L) / e_L$$

$i_o$  = current delivered by the constant current source

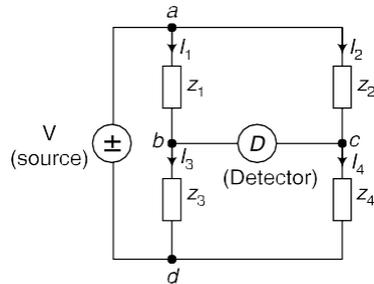
$i_L$  = current flowing through the load when it is connected to the source

$$p = (i_o - i_L) * e_L = e_L^2 * Y_o = e_L^2 / Z_o$$

Thus power extracted from the current source is small if its output admittance is small or when its output impedance is large.

# Bridges

Bridges are often used for the precision measurement of component values, like resistance, inductance, capacitance, etc. The simplest form of a bridge circuit consists of a network of four resistance arms forming a closed circuit as shown in Fig. 1. A source of current is applied to two opposite junctions and a current detector is connected to other two junctions. The bridge circuit operates on null detection principle and uses the principle of comparison measurement methods. It compares the value of an unknown component with that of an accurately known standard component. Thus, the accuracy of measurement depends on the bridge and not on the null detector. When no current flows through the null detector, the bridge is said to be balanced. The relationship between the component values of the four arms of the bridge at the balancing is called balancing condition or balancing equation. Balancing equation gives up the value of the unknown component.



**Fig. 1** General form of a bridge circuit

## 1.1.1 Advantages of Bridge Circuit

The various advantages of a bridge circuit are:

1. The balance equation is independent of the source voltage magnitude and its impedance.

## Wheatstone Bridge under Small Unbalance

As discussed in previous section different galvanometers have different current/voltage sensitivities. Hence, in order to determine whether the galvanometer has the required sensitivity to detect an unbalance condition the bridge circuit can be solved for a small unbalance by converting the Wheatstone bridge into its equivalent Thevenin's circuit.

Let us consider the bridge is balanced, when the branch resistances are  $R_1$ ,  $R_2$ ,  $R_3$  and  $R_X$ , so that at balance

$$R_X = \frac{R_1 R_3}{R_2}$$

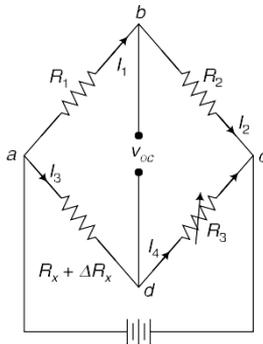
Let the resistance  $R_X$  be changed to  $\Delta R_X$  creating an unbalance. Due to this an emf  $e^1$  has appeared across the galvanometer. To compute this emf let us use the Thevenin's method. With galvanometer branch open, the open circuit voltage drop between points  $b$  and  $d$  is  $V_{oc}$ . At balance

$$E_{ab} = I_1 R_1, \quad I_1 = I_2 = \frac{E}{R_1 + R_2}$$

$$E_{ab} = \frac{E R_1}{R_1 + R_2}$$

$$E_{ad} = I_3 (R_X + \Delta R_X), \quad I_3 = I_4 = \frac{E}{R_3 + R_X + \Delta R_X}$$

$$E_{ad} = \frac{E (R_X + \Delta R_X)}{R_3 + R_X + \Delta R_X}$$



**Fig. 2** Wheatstone bridge under unbalance condition

$$V_{oc} = V_{ad} = V_{TH} = E_{ad} - E_{ab}$$

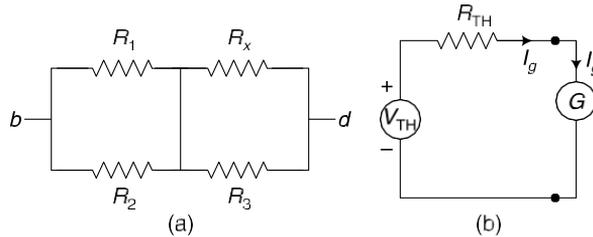
$$= E \left[ \frac{R_X + \Delta R_X}{R_3 + R_X + \Delta R_X} - \frac{R_1}{R_1 + R_2} \right]$$

$$S_B = \frac{S_V E}{\frac{R_3}{R_X} + \frac{R_X}{R_3} + 2}$$

Thus, the bridge sensitivity depends on the bridge parameter—the supply voltage and the voltage sensitivity of the galvanometer.

### Thevenin Equivalent Circuit (Under Balance Case)

The Thevenin equivalent resistance can be found by looking back into the terminal  $b-d$  and replacing the source by its internal resistance (if internal resistance is very low the source is replaced by short circuiting the terminal  $a-c$ ) as shown in Fig. (3).



**Fig. 3** (a) Equivalent resistance ( $R_{TH}$ ) between  $b$  and  $d$ , (b) Thevenin equivalent circuit for Wheatstone bridge

$$R_{TH} = (R_1 \parallel R_2) + (R_3 \parallel R_X) = \frac{R_1}{R_1 + R_2} + \frac{R_3}{R_3 + R_X}$$

and

$$V_{TH} = V_{oc} = E_{ad} - E_{ab} \quad (R_X \text{ is not changed by } \Delta R_X)$$

$$= I_2 R_X - I_1 R_1 = \frac{E R_X}{R_3 + R_X} - \frac{E}{R_1 + R_2} R_1$$

$$V_{TH} = E \left[ \frac{R_X}{R_3 + R_X} - \frac{R_1}{R_1 + R_2} \right]$$

If  $R_g$  is the galvanometer resistance and  $I_g$  is the galvanometer current, then we have

$$I_g = \frac{V_{TH}}{R_{TH} + R_g}$$

### Galvanometer Current (Unbalance Case)

Let the resistance  $R_X$  be changed by  $\Delta R_X$ , such that the arm resistance is  $R_X + \Delta R_X$  and creating an unbalance. From above Eqn.

$$V_{TH} = \frac{ER_3 \Delta R_X}{(R_3 + R_X)^2}$$

Now equivalent resistance looking back into the terminals  $b$  and  $d$ .

$$\begin{aligned} R_{eq} = R_{TH} &= (R_1 || R_2) + (R_3 || R_X + \Delta R_X) \\ &= \frac{R_3(R_X + \Delta R_X)}{R_3 + R_X + \Delta R_X} + \frac{R_1 R_2}{R_1 + R_2} \end{aligned}$$

Since  $\Delta R_X \ll R_3, R_4$ , neglecting  $\Delta R_X$  in the above equation

$$\begin{aligned} R_{TH} &= \frac{R_3 R_X}{R_3 + R_X} + \frac{R_1 R_2}{R_1 + R_2} \\ I_g &= \frac{V_{TH}}{R_{TH} + R_g} \end{aligned}$$

Now, if  $R_X = R_1 = R_2 = R_3 = R$ , we have

$$I_g = \frac{ER \Delta R_X}{\frac{4R^2}{R + R_g}}$$

$$I_g = \frac{E \Delta R_X}{4R(R + R_g)}$$

### ***Bridge Sensitivity in Terms of Current Sensitivity of Galvanometer***

Let a bridge be unbalanced by changing  $R_X$  to  $R_X + \Delta R_X$ . Due to this an emf  $e'$  has appeared across galvanometer which is basically equal to the Thevenin equivalent voltage between the galvanometer nodes. Hence, the deflection of the galvanometer

$$\theta = S_V e' = S_V V_{TH} = \frac{S_V E R_3 \Delta R_X}{(R_3 + R_X)^2}$$

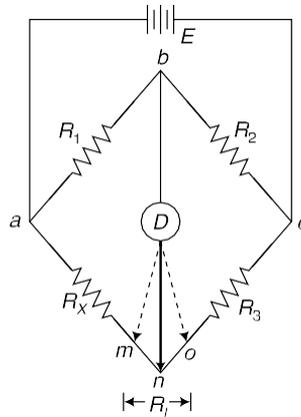
The current sensitivity of galvanometer is expressed as

$$S = \frac{S_i}{R_{TH} + R_g}$$

Hence, from equation

$$\Theta = \frac{S_i E R_3 \Delta R_X}{(R_{TH} + R_g)(R_3 + R_X)^2}$$

errors. The consideration of effect of contact and lead resistances is the basic purpose of Kelvin bridge. So that it provides increased accuracy for measurement of low value resistances Fig. 4 shows the basic circuit of Kelvin bridge.



**Fig. 4** Kelvin bridge

The resistance  $R_l$  represents the resistance of connecting leads from  $R_x$  to  $R_3$ . The  $R_x$  is unknown resistance to be measured.

Two possible galvanometer connections are indicated by dotted lines either to point  $m$  or to point  $o$ . When the galvanometer is connected to point  $m$ , the lead resistance  $R_l$  is added to the unknown resistance value resulting in high value of unknown resistance  $R_x$ . On the other hand, if the galvanometer is connected to point  $o$ , the lead resistance  $R_l$  is added to standard resistance  $R_3$ , resulting too low value of unknown resistance  $R_x$ . If the galvanometer is connected to an intermediate point say  $n$  (as shown by dark line), such that the ratio of the lead resistance from point  $m$  to  $n$  and that from  $o$  to  $n$  is equal to the ratio of  $R_1$  and  $R_2$ , i.e.,

$$\frac{R_{mn}}{R_{on}} = \frac{R_1}{R_2}$$

The Bridge balance equation is given as

$$(R_x + R_{mn}) R_2 = R_1 (R_3 + R_{on})$$

Since  $R_3$  and  $R_x$  are changed to  $R_3 + R_{on}$  and  $R_x + R_{mn}$  respectively, we

$$\begin{aligned} \frac{R_{mn}}{R_{on}} + 1 &= \frac{R_1}{R_2} + 1 \\ &= \frac{R_{mn} + R_{on}}{R_{on}} = \frac{R_1 + R_2}{R_2} \end{aligned}$$

Since  $R_l = R_{mn} + R_{on}$

Since  $R_l = R_{mn} + R_{on}$

Hence, we get

$$\frac{R_l}{R_{on}} = \frac{R_1 + R_2}{R_2}$$

$$R_{on} = \frac{R_2}{R_1 + R_2} \diamond R_l$$

Hence, 
$$R_{mn} = R_l - R_{on} = R_l \left( \frac{R_1}{R_1 + R_2} \right)$$

Substituting the value of  $R_{mn}$  and  $R_{on}$  we get

$$\left( R_x + \frac{R_l R_1}{R_1 + R_2} \right) R_2 = R_l \left( \frac{R_2 + R_2 R_l}{R_1 + R_2} \right)$$

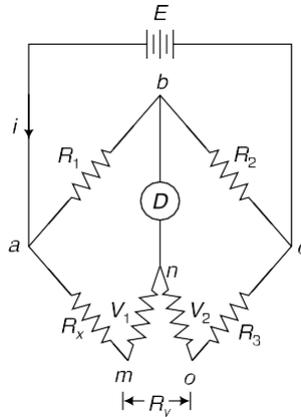
$$R_x + \frac{R_l R_1}{R_1 + R_2} = \frac{R_l R_2}{R_2} + \frac{R_l R_2}{R_1 + R_2}$$

$$R_x = \frac{R_l R_2}{R_2}$$

Above equation represents the balance equation for Wheatstone bridge and hence eliminating the resistance of connecting leads from point  $m$  to  $o$ . However, practically it is very difficult to determine the intermediate point  $n$ . The modified form of Kelvin bridge is termed Kelvin double bridge.

### 1.2.2 Kelvin Double Bridge

The modified form of Kelvin bridge is shown in Fig. 5. The two actual resistances of correct ratio are connected between point  $m$  and  $o$ . The galvanometer is connected at the junction of these resistances. Due to the introduction of second set of ratio arms, this bridge is termed Kelvin double bridge.



**Fig. 5** Kelvin double bridge

The first set of ratio arm is  $R_1$  and  $R_2$  and the second set of ratio arm is  $r_1$  and  $r_2$ . The galvanometer is connected to point  $n$  such as to eliminate the effect of connecting lead of resistance  $R_l$  between the unknown resistance  $R_X$  and standard resistance  $R_3$ . The ratio of  $r_1/r_2$  is made equal to  $R_1/R_2$ . At bridge balance condition the current through galvanometer is zero, i.e., the voltage drop between point  $a$  to  $b$  ( $E_{ab}$ ) is equal to the voltage drop between the point  $a$ - $m$ - $n$  i.e.  $E_{amn}$

$$\begin{aligned}
 E_{ab} &= \frac{R_1}{R_1 + R_2} \cdot E \\
 &= \frac{R_1}{R_1 + R_2} i [R_X + R_y \parallel (r_1 + r_2) + R_3] \\
 &= \frac{R_1 i}{R_1 + R_2} \left[ R_X + \frac{R_y (r_1 + r_2)}{R_y + r_1 + r_2} + R_3 \right]
 \end{aligned}$$

$$\begin{aligned}
E_{amn} &= E_{am} + E_{r_1} \\
&= \left[ iR_X + \frac{i r_1}{(r_1 + r_2)} \left[ \frac{(r_1 + r_2)R_y}{r_1 + r_2 + R_y} \right] \right] \\
&= iR_X + \frac{i r_1 R_y}{r_1 + r_2 + R_y} \\
&= i \left[ R_X + \frac{r_1 R_X}{R_y + r_1 + r_2} \right]
\end{aligned}$$

At bridge balance

$$E_{ab} = E_{amn}$$

$$\frac{R_1}{R_1 + R_2} \left[ R_X + R_3 + \frac{R_y(r_1 + r_2)}{(R_y + r_1 + r_2)} \right] = R_X + \frac{r_1 R_y}{R_y + r_1 + r_2}$$

$$R_X \left( \frac{R_2}{R_1 + R_2} \right) = \frac{R_1 R_3}{(R_1 + R_2)} + \frac{r_2 R_y}{R_y + r_1 + r_2} \left[ \frac{R_1 r_2 - R_2 r_1}{(R_1 + R_2) r_2} \right]$$

$$R_X = \frac{R_1 R_3}{R_2} + \frac{r_2 R_y}{R_y + r_1 + r_2} \left[ \frac{R_1}{R_2} - \frac{r_1}{r_2} \right]$$

Since  $\frac{R_1}{R_2} = \frac{r_1}{r_2}$

$$R_X = \frac{R_1 R_3}{R_2}$$

The above equation represents the standard Wheatstone bridge balance equation. Since the resistance  $r_1$ ,  $r_2$ , and  $R_y$  are not present in this equation. Hence, the effect of lead and contact resistance is completely eliminated. In typical Kelvin's bridge the range of resistance lies between 0.1 mW and 1.0 W, with accuracy of  $\pm 0.05\%$  to  $\pm 0.2\%$ .

**Example 1** Kelvin double bridge uses:

Standard resistance = 100 mW

Inner ratio arms = 15 W and 30 W

Outer ratio arms = 40 W and 60 W

If the resistance of the connecting leads from standard to unknown resistance is 800 mW. Calculate the unknown resistance under this condition.

*Solution* Given

$$R_1 = 40 \text{ W}, R_2 = 60 \text{ W}, R_3 = 100 \text{ mW}$$

$$r_1 = 15 \text{ W}, r_2 = 30 \text{ W}, R_y = 800 \text{ mW}$$

From equation

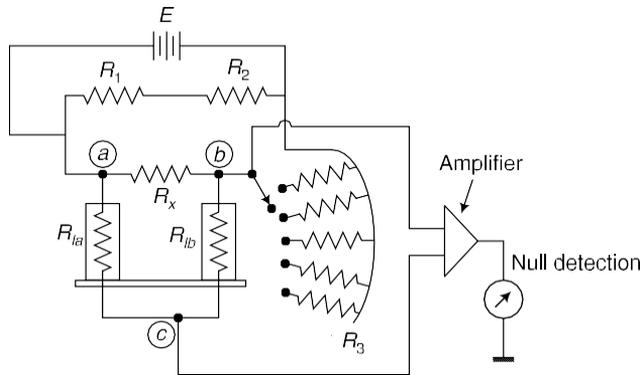
$$\begin{aligned} R_x &= \frac{R_1 R_3}{R_2} + \frac{r_2 R_y}{R_y + r_1 + r_2} \left[ \frac{R_1}{R_2} - \frac{r_1}{r_2} \right] \\ &= \frac{40 \times 100 \times 10^{-6}}{60} + \frac{30 \times 800 \times 10^{-6}}{(800 \times 10^{-6} + 15 + 30)} \left[ \frac{40}{60} - \frac{15}{30} \right] \\ &= 0.66 \times 10^{-4} + 5.33 \times 10^{-4} [0.16] \\ &= 1.5128 \times 10^{-4} \text{ W} \\ &= 151.28 \text{ mW} \end{aligned}$$

## HIGH RESISTANCE MEASUREMENT BRIDGE

The measurement of high resistance of the order of  $10^{10}$  W or more is often required in electrical equipment. For example, (i) insulation resistance of components like machines, cables, (ii) leakage resistance of capacitors, (iii) resistance of high circuit elements like vacuum tube circuits, etc. Because of very high resistance, current, flowing through the measuring circuit is low, which is

very difficult to sense. Normal Wheatstone bridge used for measurement of resistances is not suitable for this purpose due to the leakage current around the test specimen becomes of the same order as through the specimen itself or quite high compared to it.

For high resistance measurement, high voltage source is used to obtain sufficient current and galvanometer deflection. As a result leakage current is also increased. In order to avoid the leakage current effect in the measuring circuit, guard circuits are generally used. The high resistance to be measured is mounted on two insulating posts which in turn are mounted on a metal platform known as guard point. The guard point is electrically connected to the junction of the ratio arms of the bridge circuit. Due to this insulating posts become electrically parallel to the resistances of the ratio arm resistance. Hence, leakage resistance  $R_{la}$  is parallel to  $R_1$  and  $R_{lb}$  to  $R_2$ . Effectively the unknown resistance  $R_X$  to be measured has three terminals marked (a), (b) and (c) as shown in Fig. 6. With  $R_{la}$  and  $R_{lb}$  very large, resistances  $R_1$  and  $R_2$ , whom they are paralleling, are not much affected as  $R_1$  and  $R_2$  are not made very large, like  $R_3$  which again is selectable. The unknown resistance  $R_X$  is obtained with null balance method and an amplifier is used to drive the display meter which is basically a null detection.



**Fig. 6** High resistance measurement bridge

**Example 2** (a) What would be the error in measuring a high resistance of  $10^9 \text{ W}$  if the leakage resistances  $R_a$  and  $R_b$  are  $10^{10} \text{ W}$  each but the guard point arrangement is not used?

(b) If the leakage resistances are the same as the unknown resistance, what would be the error.

**Solution** (a) In the absence of the guard point arrangement, two  $10^{10} \text{ W}$  resistances are in series and become parallel to the  $10^9 \text{ W}$  resistance. Hence, the effective unknown resistance

$$R_X = \frac{10^9 \times 2 \times 10^{10}}{10^9 (1 + 20)} = 0.95 \times 10^9 \text{ W}$$

$$\text{The error} = \frac{(10^9 - 0.95 \times 10^9)}{10^9} \times 100 = 5\%$$

(b) In this case, the effective unknown resistance

$$R_X = \frac{10^9 \times 2 \times 10^9}{10^9 (1 + 2)} = 0.67 \times 10^9 \text{ W}$$

$$\begin{aligned} \text{The error} &= \frac{(10^9 - 0.67 \times 10^9)}{10^9} \times 100 \\ &= 33.3\% \end{aligned}$$

## AC BRIDGES AND THEIR APPLICATIONS

An ac bridge similar to dc bridges consists of four arms, an ac source of excitation at the desired frequency and a null detector. For measurements at low frequencies the power line may be used as a source of excitation whereas at higher frequencies generally, an oscillator is used as a source. The operating frequencies of these oscillators are constant and easily adjustable. A typical oscillator has a frequency range of 40 Hz to 125 kHz with a power output of 7W headphones, vibrational galvanometer and tuneable amplifier circuits are generally used as a null detector for ac bridges. The headphones are used as a detector at the frequency of 250 Hz to 3-4 kHz. While working with single frequency a tuned detector is the most sensitive detector. Vibrational galvanometers are useful for low audio frequency range from 5 Hz to 1 kHz, but are commonly used below 200 Hz. Tuneable amplifier detectors are used for frequency range of 10 Hz to 100 Hz.

### General Equation for Bridge Balance

Let us consider a general form of an ac bridge as shown in Fig. 7. The bridge circuit consists of a network of four impedance arms  $z_1$ ,  $z_2$ ,  $z_3$  and  $z_4$  respectively, forming a closed circuit. For bridge balance, the potential of point  $b$  must be same as the potential of point  $d$ . These potentials must be equal in terms of amplitude as well as phase. Thus, the voltage drop from  $a$  to  $b$  must be equal to voltage drop across  $a$  to  $d$ , in both magnitude and phase for the bridge balance, i.e.

$$E_1 = E_3$$

$$i_1 Z_1 = i_3 Z_3 \tag{i}$$

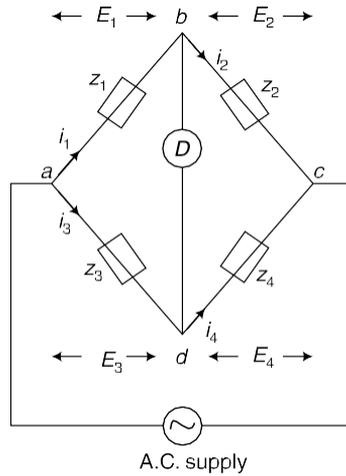
Also at balance

$$i_1 = i_2 = \frac{E}{Z_1 + Z_2}$$

and

$$i_3 = i_4 = \frac{E}{Z_3 + Z_4}$$

Hence, from Eqn. (i), we have



**Fig. 7** Basic ac bridge circuit

$$\frac{EZ_1}{Z_1 + Z_2} = \frac{EZ_3}{Z_3 + Z_4}$$

$$Z_1 Z_4 = Z_2 Z_3$$

Equation (2.32) is the equation for balance of ac bridge in the impedance form. The balance equation in the admittance (reciprocal of impedance) form can be expressed as

$$\frac{1}{Y_1 Y_4} = \frac{1}{Y_2 Y_3}$$

$$Y_1 Y_4 = Y_2 Y_3$$

In the polar form the impedance  $Z$  can be written as

$$Z = Z - \mathbf{q}$$

where  $Z$  represents the impedance and  $\mathbf{q}$  represents the phase angle of complex impedance  $Z$ . Hence, the bridge arm impedances in polar form can be expressed as

$$Z_1 = Z_1 - \mathbf{q}_1$$

$$Z_2 = Z_2 - \mathbf{q}_2$$

$$Z_3 = Z_3 - \mathbf{q}_3$$

$$Z_4 = Z_4 - \mathbf{q}_4$$

where  $Z_1, Z_2, Z_3$  and  $Z_4$  are the magnitudes and  $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3,$  and  $\mathbf{q}_4$  are the phase angles. Hence, the balance equation in polar form representation will be

$$(Z_1 - q_1) \angle (Z_4 \angle q_4) = (Z_2 - q_2) (Z_3 - q_3)$$

Since, in complex number multiplication the magnitudes are multiplied and the phase angles are added, the equation can be written as

$$Z_1 Z_4 / (q_1 + q_4) = Z_2 Z_3 / (q_2 + q_3)$$

$$Z_1 Z_4 = Z_2 Z_3$$

and 
$$-q_1 + -q_4 = -q_2 + -q_3$$

Hence, from above equation, the two conditions must be satisfied for bridge balance.

- (i) The product of the magnitudes of the opposite arms must be equal.
- (ii) The sum of phase angles of the opposite arms must be equal.

The value of phase angles depends on the type of components of individual impedance. For inductive impedance the phase angles are positive and for capacitive impedance the phase angles are negative, i.e.,

$$Z_L = R + jX_L = |Z_L| \angle + \phi$$

$$Z_C = R - jX_C = |Z_C| \angle - \phi$$

where

$$X_L = 2\pi fL$$

$$X_C = \frac{1}{2\pi fC}$$

$f$  = operating frequency

**Example 3** The four impedances of an ac bridge as shown in Fig. 2.8 are  $Z_1 = 500 \angle 40^\circ \text{ W}$ ,  $Z_2 = 100 \angle -90^\circ \text{ W}$ ,  $Z_3 = 45 \angle 20^\circ \text{ W}$ ,  $Z_4 = 30 \angle 30^\circ \text{ W}$ . Find out whether the bridge is balanced or not.

**Solution** As we know in polar co-ordinate system representation, the bridge balance conditions are

$$Z_1 Z_4 = Z_2 Z_3 \text{ and}$$

$$-q_1 + -q_4 = -q_2 + -q_3$$

Thus,

$$Z_1 Z_4 = 500 \times 30 = 15 \times 10^3 \text{ W}$$

$$Z_2 Z_3 = 100 \times 45 = 4.5 \times 10^3 \text{ W}$$

also

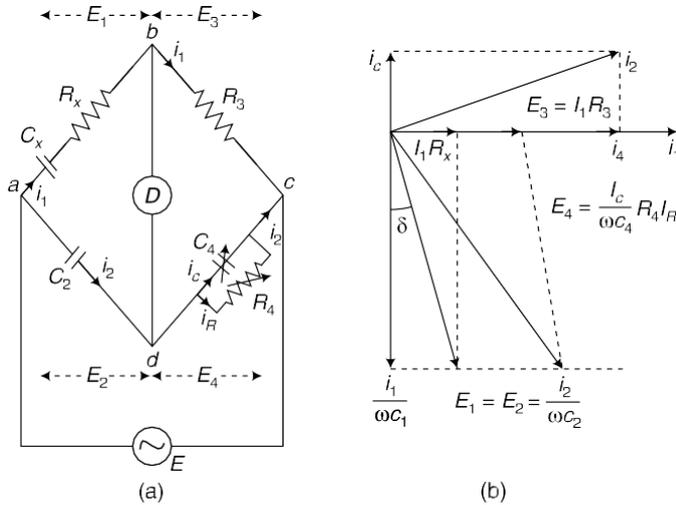
$$-q_1 + -q_4 = 40^\circ + 30^\circ = 70^\circ$$

$$-q_2 + -q_3 = 90^\circ + 20^\circ = 70^\circ$$

This indicates that phase relationship is satisfied whereas magnitude condition is not satisfied. Hence, bridge is unbalanced.

### Schering Bridge (Measurement of Capacitance)

This is the most common bridge used for measurement of unknown capacitance, dielectric loss, relative permittivity and power factor. Figure 8 shows the basic circuit arrangement of the bridge and its phasor diagram under balance conditions.



**Fig. 8** (a) Schering bridge for measurement of capacitance (b) Phasor diagram

Two branches consist of non-inductive resistance  $R_3$  and a standard capacitor  $C_2$ . The standard capacitor is usually a high-quality mica capacitor (low-loss) for general measurement or an air capacitor (having very stable value and a very small elastic field) for insulation measurement. One of the arms consists of a variable capacitor connected in parallel with a variable non-inductive resistance  $R_4$ . The remaining arm consists of unknown capacitor  $C_X$  whose capacitance is to be determined. Connected in series with a resistance  $R_X$  to represent loss in the capacitance  $C_X$ , the impedance of four arms are

$$Z_1 = \left( R_x + \frac{1}{j\omega C_x} \right)$$

$$Z_2 = \left( j\omega C_1 \right)$$

$$Z_3 = R_3$$

we obtain that the dissipation factor is the reciprocal of the quality factor  $Q$  and therefore

$$Q = \frac{1}{D}$$

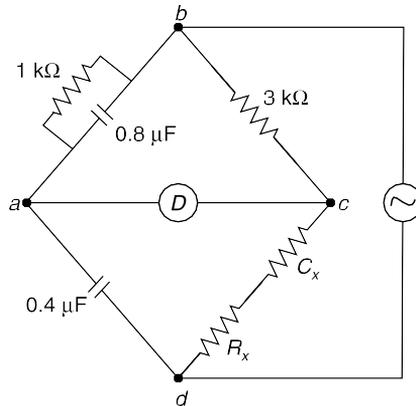
Hence, the dissipation factor tells us about the quality of the capacitor, i.e., how close the phase angle of the capacitor is to the ideal value  $90^\circ$ . Substituting the value of  $C_x$  and  $R_x$  in Eqn. (2.39), we have

$$D = w \frac{R_4 C_2}{R_3} \diamond \frac{C_4 R_3}{C_2} \quad D = w R C$$

If the frequency and resistor  $R_4$  in Schering bridge is fixed, the capacitor  $C_4$  can be calibrated to read the dissipation factor directly.

**Example 4** An ac bridge was made up as follows: arm  $ab$ , a capacitor of  $0.8 \text{ mF}$  in parallel with  $1 \text{ kW}$  resistance,  $bc$  a resistance of  $3 \text{ kW}$ , arm  $cd$  an unknown capacitor  $C_x$  and  $R_x$  in series, arm  $da$  a capacitance of  $0.4 \text{ mF}$ . The supply at  $1 \text{ kHz}$  is connected across  $bd$  and a detector across  $ac$ . Determine the value of unknown capacitance  $C_x$ , unknown series resistance  $R_x$  and dissipation factor.

**Solution** If we draw the sketch of the given problem we find the given ac bridge is Schering bridge as shown below. Hence,  $R_3 = 3 \text{ kW}$ ,  $C_2 = 0.4 \text{ mF}$ ,  $C_4 = 0.8 \text{ mF}$ ,  $R_4 = 1 \text{ kW}$ ,  $w = 1 \text{ kHz}$



$$\begin{aligned} R_x &= \frac{C_4 R_3}{C_2} \\ &= \frac{3 \times 10^3 \times 0.8 \times 10^{-6}}{0.4 \times 10^{-6}} \\ &= 6 \text{ kW} \end{aligned}$$

$$C_x = \frac{R_4 C_2}{R_3}$$

$$= 1 \times 10^3 \times 0.4 \times 10^{-6} \times 10^3 = 0.133 \times 10^{-6} \text{ F}$$

Dissipation factor  $D = \omega C_x R_x = 2\pi f C_x R_x$

$$= 2 \times 3.14 \times 10^3 \times 0.133 \times 10^{-6} \times 6 \times 10^3$$

$$= 5.011$$

### Maxwell Bridge (Measurement of Inductance)

Maxwell bridge can be used to measure inductance by comparison either with a variable standard self-inductance or with a standard variable capacitance. These two measurement methods can be done by using two different Maxwell bridge forms.

### Maxwell Inductance Capacitance Bridge

In this bridge arrangement, the value of unknown inductance is measured by comparison with a variable standard capacitor. Figure 11 shows its circuit arrangement and phasor diagram. Two arms  $bc$  and  $ad$  consist of non-inductive resistance  $R_2$  and  $R_3$ . One of the arms  $ac$  consists of variable standard capacitor  $C_4$  connected in parallel to a non-inductive resistance  $R_4$ . The remaining arm  $ab$  consists of unknown inductance  $L_x$  of effective resistance  $R_x$ . A source of current is applied to two opposite junctions across  $ac$  and a null detector is connected to the other two junctions  $b$  and  $d$ . Hence, the impedances of four arms are

$$Z_1 = R_x + j\omega L_x$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4 \parallel \frac{1}{j\omega C_4} = \frac{R_4}{1 + j\omega C_4 R_4}$$

At balance we get

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_x + j\omega L_x) \frac{R_4}{1 + j\omega C_4 R_4} = R_2 R_3$$

$$= R_x R_4 + j\omega L_x R_4 = R_2 R_3 + j\omega C_4 R_2 R_3 R_4$$

Equating real and imaginary terms, we have

$$R_x = \frac{R_2 R_3}{R_4}$$

$$L_x = R_2 R_3 C_4$$

Hence, from the above condition for bridge balance the unknown inductance value can be determined by comparison with variable standard capacitor. The quality factor of the coil is given by

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega R_2 R_3 C_4}{\left(\frac{R_2 R_3}{R_4}\right)}$$

$$Q = \omega R_4 C_4$$

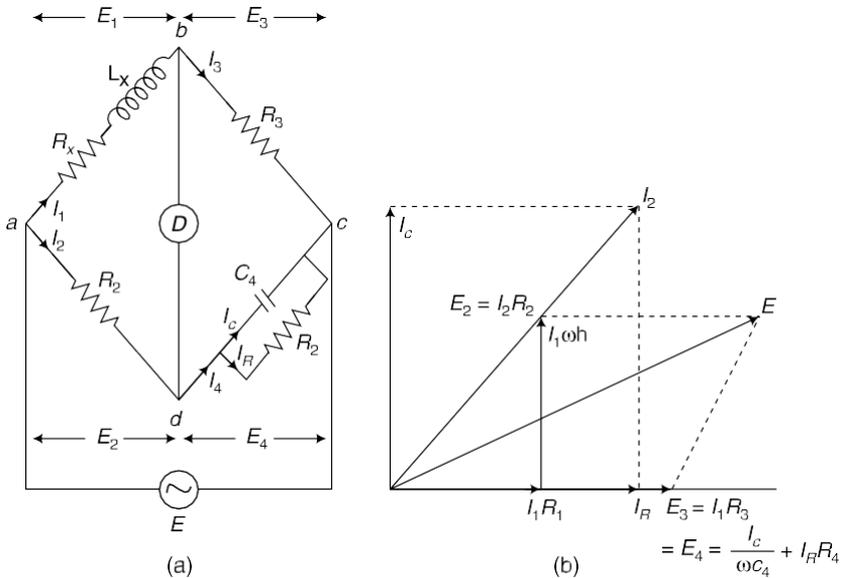


Fig. 12 (a) Maxwell inductance capacitance bridge

### Advantages

1. The balance equation is independent of losses associated with inductance.
2. The frequency does not appear in any of the two balance equations.
3. The scale of resistance can be calibrated to read inductance directly.
4. It is very useful for measurement of wide range of inductances at power and audio frequency.

## Short Answer Type Questions

1. What is a bridge circuit?
2. Explain the applications and advantages of bridge circuits?
3. What are the types of bridges?
4. Name a few ac bridges used and specify which bridge is used for what type of measurement.
5. Explain briefly how a 'Wheatstone bridge' is used for measurement of resistance.
6. Explain the sources of errors in Wheatstone bridge and its limitations.
7. Describe the sources and null detectors used for ac bridges.
8. Explain briefly the following bridges (i) Kelvin bridge (ii) Kelvin double bridge.
9. Why Maxwell Bridge is limited to the measurement of medium – Q coils?
10. Draw the circuit diagram of Schering Bridge. Derive the conditions for balancing the bridge and draw the phasor diagram during balanced condition.

## Multiple Choice Questions

1. A Wheatstone bridge cannot be used for precision measurements because errors are introduced into an account of
  - (a) thermoelectric emf
  - (b) contact resistance
  - (c) resistance of connecting leads
  - (d) all of above
2. High resistances are provided with a guard terminal. The guard terminal is used to
  - (a) guard the resistance against stray electrostatic field
  - (b) guard the resistance against overload
  - (c) bypass the leakage current
  - (d) none of above
3. Maxwell inductance capacitance bridge is used for measurement of inductance of
  - (a) low  $Q$  coils
  - (b) medium  $Q$  coils
  - (c) high  $Q$  coils
  - (d) low and medium  $Q$  coils
4. Frequency can be measured by using
  - (a) Wien bridge
  - (b) Schering bridge
  - (c) Maxwell bridge
  - (d) Heaviside Campbell bridge
5. The effective reactance of an inductive coil
  - (a) increases because of stray capacitance as frequency increases
  - (b) remains unchanged
  - (c) decreases because of stray capacitance as frequency increases
  - (d) None of above
6. A bridge circuit works at a frequency of 10 kHz. The following can be used as detector for detection of null condition.
  - (a) Vibrational galvanometer and headphones
  - (b) Headphones and tuneable amplifier
  - (c) Vibrational galvanometer and tuneable amplifier
  - (d) Vibrational galvanometer, tuneable amplifier, headphone.
7. Dissipation factor ( $D$ ) and quality factor ( $Q$ ) of a coil are related as
  - (a)  $D = Q$
  - (b)  $D = \frac{1}{Q}$
  - (c)  $D = Q^2$
  - (d) none of above
8. Inductance can be measured by
  - (a) Maxwell bridge
  - (b) Hay bridge
  - (c) Schering bridge
  - (d) Wien bridge

### References-

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2. Modern Electronic Instrumentation and Measurement Techniques: Helfrick & Cooper
3. Electrical Measurement and Measuring Instruments - Golding & Waddis